CS 418: Homework #1

Assigned: Thursday January 19, 2006

Due: Thursday January 26, 2006 at the end of class

This homework is meant to help you assess your knowledge of linear algebra and geometry. We expect that most of this should be review of material from your earlier courses. If you need a bit of a refresher for linear algebra and geometry, most of what you need to know can be found in the textbook by Edward Angel, specifically in the Appendices.

Please be organized when writing your answers to these questions. Make sure that all solutions are clearly indicated and labelled with the question they are answering. Remember to write clearly and legibly. *Unreadable answers will receive 0 credit.*

1. (15pt)
   (a) Let $\mathbf{v} = [\,5\ 3\ 7\,]$. Compute $\|\mathbf{v}\|$, the vector length of $\mathbf{v}$.
   (b) Let $\mathbf{u} = [\,2\ 3\ 1\,]$ and $\mathbf{v} = [\,5\ 3\ 7\,]$. Compute the angle $\mathbf{u}$ makes with $\mathbf{v}$. Express your answer in radians.
   (c) Let $\mathbf{u} = [\,2\ 3\ 1\,]$ and $\mathbf{v} = [\,5\ 3\ 7\,]$. Find a unit vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.
   (d) Given a vector $\mathbf{v} = [x\ y]$, show that the vector $\mathbf{u} = [-y\ x]$ is orthogonal (i.e., perpendicular) to $\mathbf{v}$.

2. (15pt) Suppose you are given matrices

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 4 \\ 0 & -4 & 3 \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

   and vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.
   (a) Compute the determinant $\det \mathbf{M}$.
   (b) Compute the inverse $\mathbf{M}^{-1}$.
   (c) Compute matrix-matrix multiplication $\mathbf{MN}$.
   (d) Compute matrix-vector multiplication $\mathbf{Mv}$.
   (e) Compute the multiplication $\mathbf{uv}^T$.

3. (15pt) Suppose you are given the following four polynomials:

$$\begin{aligned} b_1(x) &= (1-x)^3 \\ b_2(x) &= 3x(1-x)^2 \\ b_3(x) &= 3x^2(1-x) \\ b_4(x) &= x^3, \end{aligned}$$

   and the following two vectors $\mathbf{B} = \begin{bmatrix} b_1(x) \\ b_2(x) \\ b_3(x) \\ b_4(x) \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \end{bmatrix}$. There exists a matrix $\mathbf{M}$ that satisfies the equation, $\mathbf{B} = \mathbf{Mv}$. Compute $\mathbf{M}$.

4. (15pt) Suppose you are given the following matrix,

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix},$$

and vectors $\mathbf{v} = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \end{bmatrix}$, $\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \\ \mathbf{g}_4 \end{bmatrix}$, where $\mathbf{g}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{g}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{g}_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$, $\mathbf{g}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$. Compute $\mathbf{v}^T\mathbf{M}\mathbf{G}$.

5. (10pt) Suppose you are given the 2-D points $\mathbf{p}_1 = [x_0\ y_0]$ and $\mathbf{p}_2 = [x_1\ y_1]$. Show that the equation of the line between them is $(y_1 - y_0)x - (x_1 - x_0)y = y_1x_0 - x_1y_0$.

6. (15pt) The equation of a plane in 3-D space is $ax + by + cz + d = 0$. Given three points $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ find the coefficients a, b, c for the plane containing the three given points.

7. (15pt) Let the matrix $\mathbf{M}(\theta)$ be defined as follows:

$$\mathbf{M}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

   (a) A square matrix $\mathbf{M}$ is said to be *orthogonal* if and only if its inverse exists and $\mathbf{M}^{-1} = \mathbf{M}^{\mathsf{T}}$. Show that $\mathbf{M}(\theta)$ is orthogonal.

   (b) Show that $\mathbf{M}(\theta_1)\mathbf{M}(\theta_2) = \mathbf{M}(\theta_1 + \theta_2)$.

   (c) Show that, for all possible values of θ, the inverse of $\mathbf{M}(\theta)$ is $\mathbf{M}(-\theta)$.