Announcements

(1) EXAM next Tuesday (pm)

- see website for details
incl. review session
past exam(s)
etc.

Chaps 5+7
CFLs

(2) THURSDAY CLASS

HERE

1) finish up normal forms
left recursion

TODAY

2) closure properties

3) decision algorithms for CFLs

Left recursion?

$A \rightarrow Aa$

$A \rightarrow BC$

☹ BAD

Recursive Descent Parsing

$b a^*$

$A \rightarrow Aa \mid b$

$A a a a a a$

$b a a a a a$

$A \rightarrow b A'$

$A' \rightarrow \cancel{a} \mid aA' \mid \epsilon$

a^*

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \ldots \mid A\alpha_k$ left-recursive prod.

$A \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$ non left-prod.

$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots \mid \beta_n A'$

$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \ldots \mid \alpha_k A'$

$A \rightarrow aBC$

$aCDE$

$A \rightarrow a\alpha$

CFLs closed under

- union ✗
- concatenation ✗
- $*$ ✗
- homomorphism ←
- substitution ← ✗
- $\cap$ with regular languages
- reversal ✗

NOT CLOSED

- $\cap$
- complement

L_0 CFL

$0 \to \{\ ,\ ,\ ,\ \}$

$1 \to L_1$ CFL

L over Σ CFL

$\forall a \in \Sigma$ L_a is a CFL

$L =$ palindromes over $\{a,b\}^*$

$L_a = \{0^n 1^n : n \geq 0\}$ ←

$L_b = \{$even # of 0's$\}$

abba

0011 1 10110 01

start symbols

L G S_a

if L_a S_a ←

L_b S_b

$\vdots$

L_c S_c

$S \to \sim X\ a\ b\ c\ Y$

(1) closure under union

$S \to S_1 \mid S_2$

L_1 L_2

G_1 G_2

S_1 S_2

(2) closure under concat

$S \to S_1 S_2$

$L = L_1 \cdot L_2$

(3) closure under *

$S \to S_1 S \mid \epsilon$

" " +

$S \to S_1 S \mid S_1$

homomorphism h: $h(0) = aba$
$h(1) = bc$

$S \to 0S0 \mid 1S1 \mid 0$

$aba\,S\,aba \mid bc\,S\,bc \mid aba \mid bc$

$h: \Sigma \to \Delta^*$

$\Sigma = \{0, 1\}$

$\Delta = \{a, b, c\}$

$010 \in L$

aba bc aba

$h(0)\,S\,h(0) \mid h(1)\,S$

$S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

$L_a = \{0^n 1^n : \ldots\}$ $\quad G_a \quad S_a \to 0 S_a 1 \mid \epsilon$

L_b even # of 0's $\quad S_b \to \boxed{\&\, 00 \;\vdots}$

new grammar

$S \to S_a S S_a \mid S_b S S_b \mid S_a \mid S_b \mid \epsilon$

$A \to aABbC$

$\cancel{S \to abc}$

$\to cba$

$\cancel{aABbC \to a}$

$\cancel{S \to ABC} \longrightarrow S \to CBA$

$\forall$ productions

$A \to \alpha$ replace with $A \to \alpha^R$

works because $(w_1 w_2 w_3 \ldots w_k)^R$

$= (w_k)^R (w_{k-1})^R \ldots (w_2)^R (w_1)^R$

$Q_{12} = Q_1 \times Q_2$

$\delta_{12}(\langle q_1, q_2\rangle, a) = \langle \underline{\delta_1(q_1,a)}, \underline{\delta_2(q_2,a)} \rangle$

IF L_1 is CFL

L_2 is regular $\Longrightarrow L_1 \cap L_2$ is CFL

PDA $M_1 = (Q_1, \Sigma_1, \delta_1, \Gamma_1, q_0, F_1, Z_0)$

DFA $M_2 = (Q_2, \Sigma_1, \delta_2, q_0^2, F_2)$

PDA M $\quad Q = Q_1 \times Q_2$

$\delta(\langle q_1, q_2\rangle, a, Z)$

$= \{ \langle q, \alpha \rangle : \quad q = \langle p, r\rangle$

$r = \delta_2(q_2, a)$

$(p, \alpha) \in \delta_1(\underline{q_1, a, Z})$ $\}$

$L_1 \cap L_2 = \underline{\underline{L}}$ not CFL

But L_1, L_2 are CFLs.

$$L = \{0^n 1^n 2^n : n \geq 0\}$$

$$L_1 = \{0^n 1^n 2^m : n, m \geq 0\}$$

$$L_2 = \{0^m 1^n 2^n : n, m \geq 0\}$$

$$L_1 \cap L_2 = L$$

non CFL.

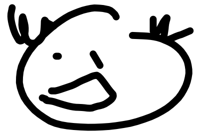
CFLs not closed under $\cap$.

$L = \{a^i b^j c^i d^j\}$

CFLs closed under union ✓

SBWOC ... " " complement

Then DeMorgan Says....



$$A \cap B = \overline{(\overline{A} \cup \overline{B})}$$

would be CFL if A, B were CFLs

Therefore, not closed under complement.

Show strings not of form $\underbrace{\{0^n 1^n 2^n \ldots\}}$

(1) is CFL.

exercise

(2) strings not of the form $\{ww \ldots\}$