

# CSE 401/CS 450/MATH 450/ECE 491: Final Guide, Fall 2009

Final Exam, 8 AM - 11 AM in 1310 DCL (A–M) and 1304 SC (N–Z).

General Advice: The exam will be multiple choice with a difficulty level similar to that of the quizzes and end of chapter review questions. A specific list of the scope of topics on the exam is included at below. In particular, the exam will be restricted to the sections listed. For your convenience the key topics, formula, and problems from each chapter are listed as well. Based on previous experience, a good strategy for studying for the final is as follows: (1) Study all prior quiz and midterm questions. (2) Understand all of the concepts given in the list at the end of this document. (3) Work through the examples in the book. (4) Be able to actually work simple problems (e.g.,  $2 \times 2$ ,  $3 \times 3$ ,  $3 \times 2$ , triangular, diagonal, etc...). (5) For each chapter: existence/uniqueness, conditioning, key methods, relative (and absolute) complexity and stability of the key methods. (6) Look over the review questions at the end of each chapter.

## [1] Sections 1.1 to 1.3.9

- Error, Accuracy, Stability:  
Absolute Error, Relative Error  
Data Error, Computational Error  
Truncation Error, Rounding Error  
Forward Error, Backward Error  
Well Conditioned, Stable, Well Posed
- Floating-Point Arithmetic:  
Underflow, Overflow, Machine Precision  
Subnormals, Gradual Underflow  
Cancellation
- Important Formula:  
Absolute Error, Relative Error  
Forward Error, Backward Error  
Condition Number  
Machine precision definitions on page 5
- Know how to compute:  
Forward/Backward Error  
Absolute/Relative Error  
 $\text{fl}(x+y)$ ,  $\text{fl}(x-y)$ ,  $\text{fl}(x*y)$ ,  $\text{fl}(x/y)$   
E.g., when does  $1 + a = 1$  ?

## [2] Sections 2.1 to 2.5.3

- Existence and Uniqueness:  
Nonsingular, Singular
- Conditioning:  
Vector and Matrix Norms  
Condition Number  
Error bounds, residual
- Solving:  
Upper and Lower Triangular  
Forward and Back Substitution  
Elementary Elimination Matrices  
Partial or Row Pivoting, Complete Pivoting  
LU Factorization, Gaussian Elimination

## Complexity of LU vs Inversion

Complexity of solving modified systems  
Symmetric, Positive Definite, Banded, Sparse  
Cholesky

Complexity for banded systems

- Important Formula:  
1-norm, 2-norm, inf-norm, p-norm for vectors  
1-norm and inf-norm for matrices  
Condition Number  
Properties of vector norms  
Properties of matrix norms  
Properties of the Condition Number  
Error bounds using condition number  
Elementary Elimination Matrices
- Know how to compute:  
Solution to triangular systems  
LU Factorizations ( $2 \times 2$  or  $3 \times 3$  matrices)  
Norms and condition numbers of diagonal matrices  
Solve small linear systems of equations

## [3] Sections 3.1 to 3.7 (exclude 3.4.2)

- Existence and Uniqueness:  
Span(A), Rank(A)  
Normal Equations  
Orthogonality of Span(A) with Residual
- Conditioning:  
Pseudoinverse and condition number  
Conditioning of the normal equations
- Solving:  
Normal Equations  
Triangular systems  
QR Factorization  
Householder, Givens  
Gram-Schmidt theory (not actual formula)  
Singular Value Decomposition

2-norm and 2-norm condition number with SVD  
Pseudoinverse with SVD  
Comparison of methods (work vs stability)

- Important Formula:  
Normal Equations, Pseudoinverse, condition number  
Error bounds involving condition number  
Householder transformation  
Givens Rotation  
2-norm (condition number) using SVD
- Know how to compute:  
Setup and solution small least-squares problem (3x2)  
Normal equations  
Householder and Givens transformations  
Norms, condition numbers given the SVD

**[4]** Sections 4.1 to 4.7 (exclude 4.2.5, 4.5.8-4.5.11, 4.6)

- Existence and Uniqueness:  
Characteristic Polynomial using determinant  
Companion Matrix  
Algebraic and Geometric multiplicity  
Invariant subspace using eigenvectors  
Diagonal, Tridiagonal, Triangular, Hessenberg  
Orthogonal, Unitary, Symmetric, Hermitian, Normal  
Which matrices are diagonalizable?  
Which matrices have real eigenvalues?  
Diagonalizable using general similarity transforms?  
Diagonalizable using orthogonal/unitary transforms?
- Conditioning:  
Bound involving cond of matrix of eigenvectors  
Bound involving angle between right/left eigenvectors
- Solving:  
Shift, Inversion, Powers, Polynomials of A  
Similarity Transformations  
Unitary and Orthogonal Similarity Transformations  
Eigenvalues of diagonal, triangular, block triangular  
Defective matrices are not diagonalizable  
Normalized power iteration with/without shifts  
Normalized inverse iteration with/without shifts  
Rayleigh quotient  
Rayleigh quotient iteration  
Deflation  
Jordan, Schur, and Real Schur forms  
QR iteration with/without shifts

Preliminary reduction to Hessenberg or Tridiagonal  
Properties of Lanczos and Arnoldi  
Comparison of methods (stability vs work)

- Important Formula:  
Characteristic Polynomial  
Bounds on p. 167 and 168  
Eigenvalues after shift, inverse, powers, polynomial  
Rayleigh quotient  
Singular values from eigenvalues of  $A^T A$  and  $A A^T$
- Know how to compute:  
Eigenvalues of a small matrix (2x2)  
Eigenvalues of a diagonal or triangular matrix  
Eigenvectors of a small matrix (2x2)  
1-step of the normalized power iteration  
1-step of inverse iteration (for triangular matrix)  
Rayleigh quotient  
Singular values of a small matrix (2x2)

**[5]** Sections 5.1 to 5.6.4

- Existence and Uniqueness:  
Scalar and Systems case  
Single and multiple roots
- Conditioning:  
Derivative and Jacobian  
Multiple roots  
Condition number
- Rates of convergence:  
Linear, superlinear, quadratic  
Number of correct digits per iteration
- Solving Scalar Problems:  
Bisection  
Stability, rate of convergence of bisection  
Fixed point iterations  
Conditions for convergence of fixed point iterations  
Rate of convergence of fixed point iterations  
Newton's method  
Secant method  
Inverse interpolation (gen. concept/props.)  
Linear fractional interpolation (gen. concept/props.)  
Safeguard methods  
Zeros of polynomials by Newton's method  
Zeros of polynomials by companion matrix  
General comparison of methods for scalar problems
- Solving Systems of Nonlinear Equations:  
Newton's Method

Convergence of a general fixed-point scheme  
Features of Broyden's method  
Trust Region and Damped Newton

- Important Formula:  
Condition number for root finding  
Convergence rate  
Convergence of (scalar/systems) fixed point iteration
- Know how to compute  
Jacobian of a system of equations  
Condition number for root finding  
1 or 2 steps of bisection  
1 step of a fixed point iteration  
1 step of Newton's method  
1 step of Secant method  
1 step of Newton's method for systems

**[6]** Sections 6.1 to 6.7.2 (excluding 6.5.7, 6.6.2, 6.7.1)

- Existence and Uniqueness:  
Global vs Local optimization  
Coercive  
Closed and Bounded  
Convex Function  
Convex Set  
First Order necessary condition  
Second Order sufficient (but not necessary) condition  
First Order necessary condition for constrained systems  
(Excluded: second order condition for constrained systems)
- Conditioning:  
Eigenvalues of Hessian  
(Excluded: sensitivity of constrained systems)
- Solving:  
Golden Section Search  
Successive Parabolic Interpolation (general idea)  
Newton's Method  
Safe Guarding  
Direct Search  
Steepest Descent  
Trust Region and Trust Radius  
BFGS (general features)  
Conjugate gradient (general features)  
Gauss-Newton (general features)  
Penalty and Barrier methods (general idea)
- Important Formula:  
Conditioning for unconstrained optimization
- Know how to compute:

One step of Newton's method for optimization  
One step of Steepest descent for a quadratic function

Hessian for an objective function  
Eigenvalues of a Hessian at a critical point.

**[7]** Sections 7.1 to 7.4.3

- Existence, Uniqueness, and Conditioning:  
Depends on the form of the interpolating function
- Solving:  
Monomial vs. Newton vs. Lagrange  
Chebyshev Points  
Oscillation problem for equally spaced points  
Hermite Cubic  
Cubic Spline  
B-Splines (basic idea)
- Important Formula:  
Bound for error for interpolating a smooth function (p 324)
- Know how to compute:  
Poly. interpolant in Monomial/Newton/Lagrange basis  
Incremental Newton  
Number of free parameters for spline interpolant  
Number of free parameters for Hermite interpolant  
"Extra conditions" for Spline and Hermite interpolants

**[8]** Sections 8.1 to 8.7 (excluding 8.3.2, 8.4.2 to 8.5, 8.6.2)

- Existence and Uniqueness:  
Bounded functions with finite number of discontinuities  
Sufficiently smooth functions are differentiable
- Conditioning:  
Integration is well conditioned for continuous functions  
Differentiation is ill conditioned for noisy data
- Solving:  
Newton-Cotes  
Trapezoid and Midpoint rule  
Gaussian Quadrature  
Progressive Quadrature  
Composite Quadrature  
Composite Midpoint and Trapezoid rule  
Adaptive quadrature (general idea)  
Forward, Backward, Centered difference approximations.  
Richardson Extrapolation
- Important Formula:

General form of bounds for error for  
Midpoint/Trapezoid

- Know how to compute:  
Composite Trapezoid/Midpoint given tabular  $f(x)$  values  
Richardson extrapolated value for a method of order  $p$

**[9]** Sections 9.1 to 9.3.8 (excluding 9.3.5)

- Existence and Uniqueness:  
Conditioning  
Linear stability conditions  
Stable vs Asymptotically Stable vs Unstable  
Non-linear stability conditions
- Solving:  
Local Error vs Global Error  
Converting a  $k$ th-order problem into first order form  
Forward Euler, Backward Euler, and Trapezoid  
Stability for Forward Euler, Backward Euler, and Trapezoid  
Stiffness  
Necessity of Implicit method for Stiff problems  
Runge-Kutta Methods (general features)  
Multistep methods (general features)  
Predictor Corrector Methods (general features)
- Important Formula:  
Stability for Forward/Backward Euler and Trapezoid  
Stability for linear and nonlinear problems
- Know how to compute:  
One step of forward Euler (scalar and systems)  
One step of backward Euler (scalar and systems)  
One step of trapezoid rule (scalar)  
Stability for a linear problem  
Stability for a method applied to a linear problem

**[10]** Sections 10.1 to 10.6

- Existence and Uniqueness and Conditioning  
Modes are coupled to boundary conditions (gen. concept)
- Definitions:  
Separated Boundary Conditions  
Linear Boundary Value Problem
- Solving:  
Steps for the Shooting Method  
Steps for the Finite Difference Method  
Steps for the Collocation Method
- Know how to compute:  
Three-point finite difference solution  
Quadratic collocation solution

**[11]** Sections 11.1 to 11.3.2

- Existence and Uniqueness and Conditioning:  
Know it's complicated
- Types of PDEs:  
Advection Equation  
Heat Equation  
Wave Equation  
Laplace Equation
- Solving:  
Characteristics of the advection equation  
Well posed boundary conditions and characteristics  
Domain of dependence and characteristics  
Features/Example of Hyperbolic PDEs  
Features/Example of Parabolic PDEs  
Features/Example of Elliptic PDEs  
Semidiscrete Methods and Method of Lines  
Fully discrete methods (e.g. Crank-Nicolson)
- Know how to compute:  
Identify PDE type given one of the four simple examples  
Identify PDE type given its features  
Characteristics of the advection equation  
Simple Solve for 2D Fully Discrete (e.g. Ex 11.2)