

Solutions for Sample Questions for Midterm 2 (CS 421 Fall 2009)

On the actual midterm, you will have plenty of space to put your answers.
Some of these questions may be reused for the exam.

1. Using the rules provided in class, give a type inference and constraint set for the following expression for

let rec fact = fun n -> if n = 0 then 1 else let r = fact (n - 1) in n * r in fact

Each step should be labeled by the inference rule used. Use capital letters for the type meta-variables. Each step should only contain that information that follows from its conclusion and the inference rule applied. Do not solve the constraints, but do give a set comprising them all. (The rules will be provided for you on the exam, if this kind of question is asked.)

Solution: (I didn't just write the tree because it wouldn't fit.)

By the let_rec rule we have

$$\frac{(1) \{ \text{fact} : B \} \vdash (\text{fun } n \rightarrow \text{if } n = 0 \text{ then } 1 \text{ else let } r = \text{fact } (n - 1) \text{ in } n * r) : B \quad (2) \{ \text{fact} : B \} \vdash \text{fact} : A}{\{ \} \vdash \text{let rec fact = fun } n \rightarrow \text{if } n = 0 \text{ then } 1 \text{ else let } r = \text{fact } (n - 1) \text{ in } n * r \text{ in fact} : A}$$

We have (2) by the variable rule is valid given the constraint (A = B).

$$(2) \{ \text{fact} : B \} \vdash \text{fact} : A$$

By the fun rule we have

$$(3) \{ n : C, \text{fact} : B \} \vdash (\text{if } n = 0 \text{ then } 1 \text{ else let } r = \text{fact } (n - 1) \text{ in } n * r) : D$$

provided (B = C -> D)

$$(1) \{ \text{fact} : B \} \vdash (\text{fun } n \rightarrow \text{if } n = 0 \text{ then } 1 \text{ else let } r = \text{fact } (n - 1) \text{ in } n * r) : B$$

By the if_then_else rule we have

$$\frac{(4) \{ n : C, \text{fact} : B \} \vdash (n = 0) : \text{bool} \quad (5) \{ n : C, \text{fact} : B \} \vdash 1 : D \quad (6) \{ n : C, \text{fact} : B \} \vdash (\text{let } r = \text{fact } (n - 1) \text{ in } n * r) : D}{(3) \{ n : C, \text{fact} : B \} \vdash (\text{if } n = 0 \text{ then } 1 \text{ else let } r = \text{fact } (n - 1) \text{ in } n * r) : D}$$

(5) is valid by the rule for constants, given the constraint that (D = int).

$$(5) \{ n : C, \text{fact} : B \} \vdash 1 : D$$

By the rule for binary relations we have

$$\frac{(7) \{ n : C, \text{fact} : B \} \vdash n : \text{int} \quad (8) \{ n : C, \text{fact} : B \} \vdash 0 : \text{int}}{(4) \{ n : C, \text{fact} : B \} \vdash (n = 0) : \text{bool}}$$

(7) is valid by the rule for variables, provided (C = int).

$$(7) \{ n : C, \text{fact} : B \} \vdash n : \text{int}$$

(8) is valid by the rule for constants.

$$\frac{}{(8) \{n : C, \text{fact} : B\} \vdash 0 : \text{int}}$$

By the rule for let, we have

$$\frac{(9) \{n : C, \text{fact} : B\} \vdash \text{fact}(n - 1) : E \quad (10) \{r : E, n : C, \text{fact} : B\} \vdash (n * r) : D}{(6) \{n : C, \text{fact} : B\} \vdash (\text{let } r = \text{fact}(n - 1) \text{ in } n * r) : D}$$

By the rule for applications we have

$$\frac{(11) \{n : C, \text{fact} : B\} \vdash \text{fact} : F \rightarrow E \quad (12) \{n : C, \text{fact} : B\} \vdash (n - 1) : F}{(9) \{n : C, \text{fact} : B\} \vdash \text{fact}(n - 1) : E}$$

(11) is valid by the variable rule, provided $(B = F \rightarrow E)$.

$$\frac{}{(11) \{n : C, \text{fact} : B\} \vdash \text{fact} : F \rightarrow E}$$

Provided $(F = \text{int})$, by the rule for binary operations, we have

$$\frac{(13) \{n : C, \text{fact} : B\} \vdash n : \text{int} \quad (14) \{n : C, \text{fact} : B\} \vdash 1 : \text{int}}{(12) \{n : C, \text{fact} : B\} \vdash (n - 1) : F}$$

(13) is valid by the variable rule, given the constraint $(C = \text{int})$.

$$\frac{}{(13) \{n : C, \text{fact} : B\} \vdash n : \text{int}}$$

(14) is valid by the constant rule.

$$\frac{}{(14) \{n : C, \text{fact} : B\} \vdash 1 : \text{int}}$$

Thus (12) is valid. Thus (9) is valid. We have (10) left. By the rule for binary operations, provided $(D = \text{int})$ we have:

$$\frac{(15) \{r : E, n : C, \text{fact} : B\} \vdash n : \text{int} \quad (16) \{r : E, n : C, \text{fact} : B\} \vdash r : \text{int}}{(10) \{r : E, n : C, \text{fact} : B\} \vdash (n * r) : D}$$

(15) is valid by the variable rule if $(C = \text{int})$, and (16) is valid by the variable rule if $(E = \text{int})$.

$$\frac{}{(15) \{r : E, n : C, \text{fact} : B\} \vdash n : \text{int}} \quad \frac{}{(16) \{r : E, n : C, \text{fact} : B\} \vdash r : \text{int}}$$

Hence (10) is valid. Hence (6) is valid. Hence (3) is valid, and hence (1) is valid. The total constraints required for the validity of each step is $\{(A = B), (B = C \rightarrow D), (D = \text{int}), (C = \text{int}), (B = F \rightarrow E), (C = \text{int}), (D = \text{int}), (D = \text{int}), (E = \text{int})\}$. Duplicates may be removed.

2. Give a (most general) unifier for the following unification instance. Capital letters denote variables of unification. Show your work by listing the operation performed in each step of the unification and the result of that step.

$$X = f(g(x),W), h(y) = Y, f(Z,x) = f(Y,W)\}$$

Solution:

- $\{X = f(g(x),W), h(y) = Y, f(Z,x) = f(Y,W)\}$
 $\rightarrow \{h(y) = Y, f(Z,x) = f(Y,W)\}$ with $\{X = f(g(x),W)\}$ by eliminate
 $\rightarrow \{Y = h(y), f(Z,x) = f(Y,W)\}$ with $\{X = f(g(x),W)\}$ by orient
 $\rightarrow \{f(Z,x) = f(h(y),W)\}$ with $\{X = f(g(x),W), Y = h(y)\}$ by eliminate
 $\rightarrow \{Z = h(y), x=W\}$ with $\{X = f(g(x),W), Y = h(y)\}$ by decompose
 $\rightarrow \{x = W\}$ with $\{X = f(g(x),W), Y = h(y), Z = h(y)\}$ by eliminate
 $\rightarrow \{W = x\}$ with $\{X = f(g(x),W), Y = h(y), Z = h(y)\}$ by orient
 \rightarrow answer: $\{X = f(g(x),x), Y = h(y), Z = h(y), W = x\}$ by eliminate

3. For each of the following descriptions, give a regular expression over the alphabet $\{a,b,c\}$, and a regular grammar that generates the language described.

- a. The set of all strings over $\{a, b, c\}$, where each string has at most one **a**

Solution: $(b \vee c)^*(a \vee \epsilon) (b \vee c)^*$
 $\langle S \rangle ::= b \langle S \rangle \mid c \langle S \rangle \mid a \langle NA \rangle \mid \epsilon$
 $\langle NA \rangle ::= a \langle NA \rangle \mid b \langle NA \rangle \mid c \langle NA \rangle \mid \epsilon$

- b. The set of all strings over $\{a, b, c\}$, where, in each string, every **b** is immediately followed by at least one **c**.

Solution: $(a \vee c)^*(bc(a \vee c))^*$
 $\langle S \rangle ::= a \langle S \rangle \mid c \langle S \rangle \mid b \langle C \rangle \mid \epsilon$
 $\langle C \rangle ::= c \langle S \rangle$

- c. The set of all strings over $\{a, b, c\}$, where every string has length a multiple of four.

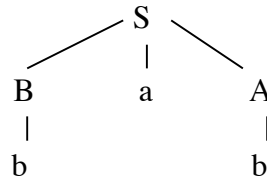
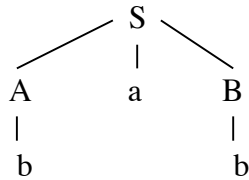
Solution: $((a \vee b \vee c) (a \vee b \vee c) (a \vee b \vee c) (a \vee b \vee c))^*$
 $\langle S \rangle ::= a \langle TH \rangle \mid b \langle TH \rangle \mid c \langle TH \rangle \mid \epsilon$
 $\langle TH \rangle ::= a \langle TW \rangle \mid b \langle TW \rangle \mid c \langle TW \rangle$
 $\langle TW \rangle ::= a \langle O \rangle \mid b \langle O \rangle \mid c \langle O \rangle$
 $\langle O \rangle ::= a \langle S \rangle \mid b \langle S \rangle \mid c \langle S \rangle$

4. Consider the following grammar:

$\langle S \rangle ::= \langle A \rangle \mid \langle A \rangle \langle S \rangle$
 $\langle A \rangle ::= \langle Id \rangle \mid (\langle B \rangle$
 $\langle B \rangle ::= \langle Id \rangle \mid \langle Id \rangle \langle B \rangle \mid (\langle B \rangle$
 $\langle Id \rangle ::= 0 \mid 1$

For each of the following strings, give a parse tree for the following expression as an $\langle S \rangle$, if one exists, or write “No parse” otherwise:

Solution: String: bab



6. Write an unambiguous grammar generating the set of all strings over the alphabet $\{0, 1, +, -\}$, where $+$ and $-$ are infix operators which both associate to the left and such that $+$ binds more tightly than $-$.

Solution:

```

<S> ::= <plus> | <S> - <plus>
<plus> ::= <id> | <plus> + <id>
<id> ::= 0 | 1
  
```

7. Write a recursive descent parser for the following grammar:

```

<S> ::= <N> % <S> | <N>
<N> ::= g <N> | a | b
  
```

You should include a datatype **token** of tokens input into the parser, one or more datatypes representing the parse trees produced by parsing (the abstract syntax trees), and the function(s) to produce the abstract syntax trees. Your parser should take a list of tokens as input and generate an abstract syntax tree corresponding to the parse of the input token list.

Solution:

```

type token = ATk | BTk | GTk | PercentTk
type s = Percent of (n * s) | N_as_s n
and n = G of n | A | B
  
```

```

let rec s_parse tokens =
  match n_parse tokens with (n, tokens_after_n) ->
    (match tokens_after_n with PercentTk::tokens_after_percent ->
      (match s_parse tokens_after_percent
        with (s, tokens_after_s) -> (Percent (n,s), tokens_after_s))
      | _ -> (N_as_s n, tokens_after_n))
  and n_parse tokens =
    match tokens
    with GTk::tokens_after_g ->
      (match n_parse tokens_after_g
        with (n, tokens_after_n) -> (G n, tokens_after_n))
  
```

```

| ATk::tokens_after_a -> (A, tokens_after_a)
| Btk::tokens_after_b -> (B, tokens_after_b)

```

```

let parse tokens =
  match s_parse tokens
  with (s, []) -> s
       | _ -> raise (Failure "No parse")

```

8. Why don't we ever get shift/shift conflicts in LR parsing?

Solution: The shift action means, when in a given state prescribing the shift, to remove the token from the top of the token stream and place it on top of the stack and move to the new state prescribe for the given state and the moved token. There is only one token stream, only one stack and the state to which to go is entirely determined by the given state and the token moved. Thus, there is only one way to execute a shift so we never have two different shifts between which to choose.

9. Consider the following grammar with terminals *, f, x, and y, and eol for “end of line”, and non-terminals S, E and N and productions

```

(P1) S => E eol
(P2) E => E * N
(P3) E => N
(P4) N => f N
(P5) N => x
(P6) N => y

```

The following are the Action and Goto tables generated by YACC for the above grammar:

STATE	ACTION					GOTO		
	*	f	x	y	eol	S	E	N
1		s5	s3	s4		2	6	7
2					acc			
3	r5	r5	r5	r5	r5	r5		
4	r6	r6	r6	r6	r6	r6		
5		s5	s3	s4				8
6	S9				s10			
7	r3	r3	r3	r3	r3	r3		
8	r4	r4	r4	r4	r4	r4		
9		s5	s3	s4				11
10	r1	r1	r1	r1	r1	r1		
11	r2	r2	r2	r2	r2	r2		

where si is shift and stack state i , rj is reduce using production Pj , acc is accept. The blank cells should be considered as labeled with error. The empty “character” represents end of input.

Describe how the sentence $fx*y\langle eol \rangle$ would be parsed with an LR parser using this table. For each step of the process give the parser action (shift/reduce), input and stack state.

Solution: In the table below, the top of the stack is on the left

Curr State	Current Stack :	Curr String	Action
		$fx*y\langle eol \rangle$	Init stack and state
st1	st1	$fx*y\langle eol \rangle$	Shift f to stack, go to state 5
st5	st1 : f : st5	$x*y\langle eol \rangle$	Shift x, go to state 3
st3	st1 : f : <u>st5</u> : x : st3	$*y\langle eol \rangle$	Reduce by prod 5: $N \Rightarrow x$, ie remove st3 and x from the stack, temporarily putting us in st5, push N and st8 (because $GOTO(st5, N) = st8$ onto stack go to state 8
st8	<u>st1</u> : f st5 : N : st8	$*y\langle eol \rangle$	Reduce by prod 4: $N \Rightarrow fN$, go to state 7
st7	<u>st1</u> : N : st7	$*y\langle eol \rangle$	Reduce by prod 3: $E \Rightarrow N$, go to state 6
st6	st1 : E : st6	$*y\langle eol \rangle$	Shift *, go to state 9
st9	st1 : E : st7 : * : st9	$y\langle eol \rangle$	Shift y, go to state 4
st4	st1 : E : st7 : * : <u>st9</u> : y st4	$\langle eol \rangle$	Reduce by prod 6: $N \Rightarrow y$, go to state 11
st12	<u>st1</u> : E : st7 : * : st9 : N : st11	$\langle eol \rangle$	Reduce by prod 2: $E \Rightarrow E*N$, go to state 6
st7	st1 : E : st6	$\langle eol \rangle$	Shift $\langle eol \rangle$, go to state 10
st11	<u>st1</u> : E : st6 : $\langle eol \rangle$: st10		Reduce by prod 1: $S \Rightarrow E\langle eol \rangle$, go to state 2
st2	st1 : S : st2		Accept