

CSE 401/CS 450/MATH 450/ECE 491: Midterm Guide, Fall 2008

Midterm Exam, Friday, October 17th, 2008, 9:00 – 9:50 AM, 1320 DCL

General Advice: The exam will be multiple choice with a difficulty level similar to that of the quizzes and end of chapter review questions. A specific list of the scope of topics on the exam is included at below. In particular, the exam will be restricted to the sections listed. For your convenience the key topics, formula, and problems from each chapter are listed as well. Based on previous experience, a good strategy for studying for the midterm is as follows: (1) Study all prior quiz questions. (2) Understand all of the concepts given in the list at the end of this document. (3) Work through the examples in the book. (4) Be able to actually work simple problems (e.g., 2×2 , 3×3 , 3×2 , triangular, diagonal, etc...). (5) For each chapter: existence/uniqueness, conditioning, key methods, relative (and absolute) complexity and stability of the key methods. (6) Look over the review questions at the end of each chapter.

[1] Sections 1.1 to 1.3.9

- Error, Accuracy, Stability:
Absolute Error, Relative Error
Data Error, Computational Error
Truncation Error, Rounding Error
Forward Error, Backward Error
Well Conditioned, Stable, Well Posed
- Floating-Point Arithmetic:
Underflow, Overflow, Machine Precision
Subnormals, Gradual Underflow
Cancellation
- Important Formula:
Absolute Error, Relative Error
Forward Error, Backward Error
Condition Number
Machine precision definitions on page 5
- Know how to compute:
Forward/Backward Error
Absolute/Relative Error
 $\text{fl}(x+y)$, $\text{fl}(x-y)$, $\text{fl}(x*y)$, $\text{fl}(x/y)$
E.g., when does $1 + a = 1$?

[2] Sections 2.1 to 2.5.3

- Existence and Uniqueness:
Nonsingular, Singular
- Conditioning:
Vector and Matrix Norms
Condition Number
Error bounds, residual
- Solving:
Upper and Lower Triangular
Forward and Back Substitution
Elementary Elimination Matrices
Partial or Row Pivoting, Complete Pivoting
LU Factorization, Gaussian Elimination
Complexity of LU vs Inversion

- Complexity of solving modified systems
Symmetric, Positive Definite, Banded, Sparse
Cholesky
Complexity for banded systems
- Important Formula:
1-norm, 2-norm, inf-norm, p-norm for vectors
1-norm and inf-norm for matrices
Condition Number
Properties of vector norms
Properties of matrix norms
Properties of the Condition Number
Error bounds using condition number
Elementary Elimination Matrices
- Know how to compute:
Solution to triangular systems
LU Factorizations (2×2 or 3×3 matrices)
Norms and condition numbers of diagonal matrices
Solve small linear systems of equations

[3] Sections 3.1 to 3.7 (exclude 3.4.2)

- Existence and Uniqueness:
Span(A), Rank(A)
Normal Equations
Orthogonality of Span(A) with Residual
- Conditioning:
Pseudoinverse and condition number
Conditioning of the normal equations
- Solving:
Normal Equations
Triangular systems
QR Factorization
Householder, Givens
Gram-Schmidt theory (not actual formula)
Singular Value Decomposition
2-norm and 2-norm condition number with SVD
Pseudoinverse with SVD

- Comparison of methods (work vs stability)
- Important Formula:
Normal Equations, Pseudoinverse, condition number
Error bounds involving condition number
Householder transformation
Givens Rotation
2-norm (condition number) using SVD
- Know how to compute:
Setup and solution small least-squares problem (3x2)
Normal equations
Householder and Givens transformations
Norms, condition numbers given the SVD

- Important Formula:
Characteristic Polynomial
Bounds on p. 167 and 168
Eigenvalues after shift, inverse, powers, polynomial
Rayleigh quotient
Singular values from eigenvalues of $A^T A$ and AA^T
- Know how to compute:
Eigenvalues of a small matrix (2x2)
Eigenvalues of a diagonal or triangular matrix
Eigenvectors of a small matrix (2x2)
1-step of the normalized power iteration
1-step of inverse iteration (for triangular matrix)
Rayleigh quotient
Singular values of a small matrix (2x2)

[4] Sections 4.1 to 4.7 (exclude 4.2.5, 4.5.8-4.5.11, 4.6) [5] Sections 5.1 to 5.6.1 (remaining after midterm)

- Existence and Uniqueness:
Characteristic Polynomial using determinant
Companion Matrix
Algebraic and Geometric multiplicity
Invariant subspace using eigenvectors
Diagonal, Tridiagonal, Triangular, Hessenberg
Orthogonal, Unitary, Symmetric, Hermitian, Normal
Which matrices are diagonalizable? have real eigenvalues?
How close to diagonal using general similarity transforms?
How about with orthogonal transforms? Unitary transforms?
- Conditioning:
Bound involving cond of matrix of eigenvectors
Bound involving angle between right/left eigenvectors
- Solving:
Shift, Inversion, Powers, Polynomials of A
Similarity Transformations
Unitary and Orthogonal Similarity Transforms
Eigenvalues of diagonal, triangular, block triangular
Defective matrices are not diagonalizable
Normalized power iteration with/without shifts
Normalized inverse iteration with/without shifts
Rayleigh quotient
Rayleigh quotient iteration
Deflation
Jordan, Schur, and Real Schur forms
QR iteration with/without shifts
Preliminary reduction to Hessenberg or Tridiagonal
Properties of Lanczos and Arnoldi
Comparison of methods (stability vs work)

- Existence and Uniqueness:
Scalar and Systems case
Single and multiple roots
- Conditioning:
Derivative and Jacobian
Multiple roots
Condition number
- Rates of convergence:
Linear, superlinear, quadratic
Number of correct digits per iteration
- Solving Scalar Problems:
Bisection
Stability, rate of convergence of bisection
Fixed point iterations
Conditions for convergence of fixed point iterations
Rate of convergence of fixed point iterations
Newton's method
Secant method
Inverse interpolation (general concept/properties)
Linear fractional interpolation (general concept/properties)
Safeguard methods
Zeros of polynomials by Newton's method
Zeros of polynomials by companion matrix
General comparison of methods for scalar problems
- Important Formula:
Condition number for root finding
Convergence rate
Convergence of fixed point iteration
- Know how to compute
Jacobian of a system of equations
Condition number for root finding
1 or 2 steps of bisection
1 step of a fixed point iteration
1 step of Newton's method (or Secant method)