

# CS 373: Theory of Computation

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University of Illinois, Urbana-Champaign

Fall 2008

# Part I

## Administrivia

# Instructional Staff

- **Instructors:**
  - Manoj Prabhakaran (mmp)
  - Mahesh Viswanathan (vmahesh)
- **Teaching Assistants:**
  - Micah Hodosh (mhodosh2)
  - Pavithra Prabhakar (pprabha2)
  - Aparna Sundar (sundar2)
- **Office Hours:**
  - Manoj: Tuesday 13:45 – 14:45, and by appointment
  - Mahesh: Thursday 10:00 – 11:00, and by appointment
  - TAs: To be announced; see course webpage

# Electronic Bulletin Boards

- **Webpage:** `www.cs.uiuc.edu/class/fa08/cs373`
- **Newsgroup:** `uiuc.class.cs373`

# Textbooks

- **Prerequisites:** All material in CS 173, and CS 225
- **Text-book:** Introduction to the Theory of Computation by Michael Sipser
- **Lecture Notes:** Available on the web-page after every class
- **Additional References**
  - Introduction to Automata Theory, Languages, and Computation: Hopcroft, Motwani, and Ullman
  - Elements of the Theory of Computation: Lewis, and Papadimitriou
  - Computers and Intractability: Garey and Johnson

# Grading Policy: Overview

## Total Grade and Weight

- **Homeworks:** 20%
- **Self Assessment Tests:** 10%
- **Midterms:** 40% ( $2 \times 20$ )
- **Finals:** 30%

# Homeworks

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- Read Homework Guidelines on course website

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- We will announce which groups will present orally

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  - Everyone gets full points for completing the test

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- Final Exam will test **all** the course material

## Part II

# Theory of Computation

# Computation

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- *Same machine for all instances*

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- Through the computational lens, mathematical problems look fundamentally different
- Not all problems are equally easy to solve
  - More *complex* problems take longer, no matter how clever you are!
- Not all problems can be solved!
  - Even when the solution is well-defined mathematically!

## Three Tales of Computation ...

# Strands of the Tale

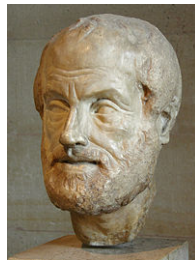
- What is the nature of infinity?
- What enables humans to communicate through language?
- How does the human brain work?

## The first story

Challenges of the Infinite  
*One machine to solve them all...*

## Infinitum actu non datur

There are no *actual* infinities; only *potential* infinities.

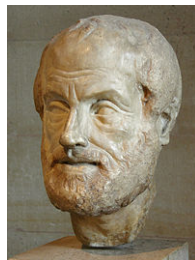


Aristotle

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*"I protest against the use of infinite magnitude as something completed, which is never permissible in mathematics. Infinity is merely a way of speaking, the true meaning being a limit which certain ratios approach indefinitely close, while others are permitted to increase without restriction." : Gauss*



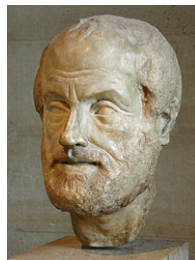
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*"There are more primes than in any given collection of prime numbers":  
Euclid*



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- Laid the foundations of the theory of infinite sets



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- Developed the theory of infinite ordinals (numbers)
- Showed how the size of (infinite) sets could be measured
  - Showed there were more real numbers than natural numbers
  - Presaged ideas that would later show that very few problems can actually be solved computationally



Georg Cantor

# Devil

- Cantor's work received widespread opposition during his time
  - Theologians saw Cantor's work as a challenge to the uniqueness of absolute infinity in the nature of God
  - Poincaré called it the “great disease” infecting mathematics
  - Kronecker called Cantor a “charlatan”, a “renegade”, and a “corrupter of youth”!

## Devil or Messiah?

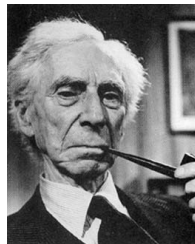
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  - Kronecker called Cantor a “charlatan”, a “renegade”, and a “corrupter of youth”!
- But, it also had some supporters

*“No one shall expel us from the Paradise created by Cantor”:* Hilbert

# Crisis in Set Theory

## Bertrand Russell (1872–1970)

- Mathematician, philosopher, writer, and political activist who won the Nobel prize in literature!
- Discovered disturbing paradoxes in Cantor's theory



Bertrand Russell

# Russell's Paradox

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- Russell: “Consider the set  $A$  of all sets that are not members of themselves. Is  $A$  a member of itself?”

# Axiomatic Method to the Rescue

David Hilbert (1863–1943)

- Solution to the crises: Formalism that avoids the paradoxes



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# Axiomatic Method to the Rescue

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- Solution to the crises: Formalism that avoids the paradoxes
  - Define concepts precisely
  - Define **axioms** and **rules of inference** that can be used to write down formal proofs



David Hilbert

# Formal Proofs

## Euclid of Alexandria (around 300 BCE)

- Euclid's Elements sets out
  - Axioms (or postulates), which are *self evident truths*, and
  - Proves all results in geometry from these truths formally



Euclid of Alexandria

## Euclid's Postulates

- A1 A straight line can be drawn from any point to any point.
- A2 A finite line segment can be extended to an infinite straight line.
- A3 A circle can be drawn with any point as center and any given radius.
- A4 All right angles are equal.
- A5 If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which the angles are less than two right angles.

# Example of a Formal Proof

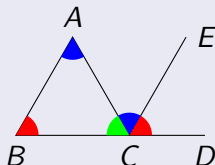
Elements: Proposition 32

## Proposition

*The interior angles of a triangle sum to two right angles.*

## Proof.

- 1 Extend one side (say) BC to D [A2]
- 2 Draw a line parallel to AB through point C; call it CE [P31]
- 3 Since AB is parallel to CE,  $\angle BAC = \angle ACE$  and  $\angle ABC = \angle ECD$  [P29]
- 4 Thus, the sum of the interior angles =  
 $\angle ACB + \angle ACE + \angle ECD = 180^\circ$



# Consistency and Completeness

## Proof System

Precise definition of what constitutes a proof — each line is an axiom, or is derived from previous lines by rule of inference.

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## Completeness

A proof system is **complete** if every true statement has a proof that adheres to its rules.

# Agenda for the 20th Century

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- Suggested 23 open problems to be investigated in the 20th century; some remain open to this day!
- One was to obtain a consistent and complete proof system for mathematics — axioms and rules that will allow all (and only) mathematical truths to be proved

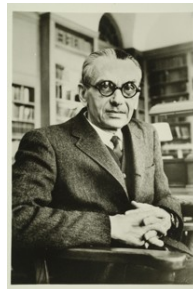
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Kurt Gödel (1906–1978)

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- At one of the satellite conferences of the same meeting, Kurt Gödel pronounced that Hilbert’s program was fated to fail!



Kurt Gödel

# Gödel's Insight

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- Proof is a hacker's dream!

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- What are limits of mechanical computation?

# The Computer Revolution

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Emil Post



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- Church ( $\lambda$ -calculus), Post (Post's machine), Turing (Turing machine) independently come up with formal definition of mechanical computation that are equivalent
- Discovered problems that cannot be solved computationally

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- The sentence itself has only 81 characters, and describes a number that cannot be described by less than 90 characters!

# Busy Beaver Function

- **Simple Program:** A program (in say  $C$ ) that takes no inputs, computes, prints some number and stops.
- **Busy Beaver Function:**  $BB(n)$  is the largest number printed by a simple program of less than  $n$  bytes

# An uncomputable function!

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- Size of above program  $\leq m + 5000 + \log m$  but prints a number  $> BB(5000 + 2 * m)$ ! Contradiction!

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- Suppose there is a consistent proof system that for every value of  $n$  proves that  $BB(n) = k$  for some  $k$ .
- The function  $\text{comp}BB(n)$  simply goes through all strings one by one, till it comes across a string that is a correct proof of  $BB(n) = k$ .

## The second story

Understanding Language  
*One grammar to generate it all...*

# The Problem of Language Acquisition

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- Language is an important human cognitive process that allows us to share information, thoughts and subjective experiences
- Though language is complex, it is acquired and used skillfully by children
- What is the mechanism behind its acquisition and use?

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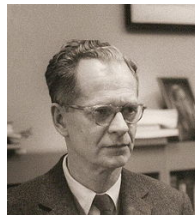
# Behaviorism

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- Behaviors are the only measurable things
- Scientific description of such behavior should not rely on internal physiological events or hypothetical constructs such as the mind

# Verbal Behavior

Burrhus Skinner (1904–1990)

- The child's mind is a blank slate, and language is learned
- The learning process is a gradual change based on sensory input provided to the organism
- Thinking is a form of “verbal behavior”

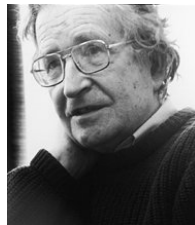


B.F. Skinner

# Critique of Verbal Behavior

Noam Chomsky (1928–)

Behaviorist account is flawed because the underpinnings of natural language are highly abstract principles, and children acquire language without explicit instruction or environment clues to these principles.



Noam Chomsky

# Humboldt: "infinite use of finite media"

Too many sentences to learn

*"A person capable of producing sentences with up to 20 words, can deal with at least  $10^{20}$  sentences. At the rate of 5 seconds per sentence, she would need 100 trillion years (with no time for eating or sleeping) to memorize them.": Pinker*

# Humboldt: “infinite use of finite media”

## Recursive build-up

- The longest English sentence in the Guinness Book of World Records is a 1,300 word sentence in Faulkner's Absalom, Absalom! that begins “They both bore it as though ...”

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- But that could be easily broken by: Pinker wrote that Faulkner wrote, “They both ...”

# Grammatical Correctness independent of Cognition

## Ungrammatical Comprehensible Sentences

- The child seems sleeping.
- It's flying finches, they are.
- Sally poured the glass with water.
- Who did a book about impress you?
- This sentence no verb.

# Grammatical Correctness independent of Cognition

## Grammatical Incomprehensible Sentences

- Chomsky: “Colorless green ideas sleep furiously.”
- Edward Lear: “It’s a fact the whole word knows, That Pobbles are happier without their toes.”

# Generative and Universal Grammars

## Chomsky's Solution

The key to language acquisition is learning a **Generative Grammar** for the language that describes

- Word categories (like nouns, verbs, etc.)
- Rules determining how categories are put together

Chomsky's theory: A core **Universal Grammar** is innate to all humans.

# Chomsky Hierarchy

Chomsky found grammars of different complexity conveniently describe various aspects of languages.

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- Lexical tokens described using “regular expressions”
- Language syntax described using “context-free grammars”

## The third story

### Understanding the Brain *A fruitful attempt*

# The Human Brain

- Central organ in the body that controls and regulates all human activity
- Believed the seat of “higher mental activity”: thought, reason and abstraction
- How does it work?

# Neurons

Santiago Ramón y Cajal (1852–1934)

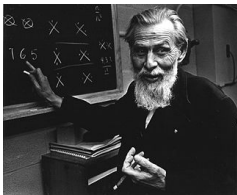
- Neurons are the primary functional unit of the brain and the central nervous system
- Neurons receive information at **dendrites** and transmit via **axons**
- They communicate with each other through junctions called **synapses**



Santiago Ramón y  
Cajal

# Mathematical Model of Neural Nets

McCullough and Pitts (1943)



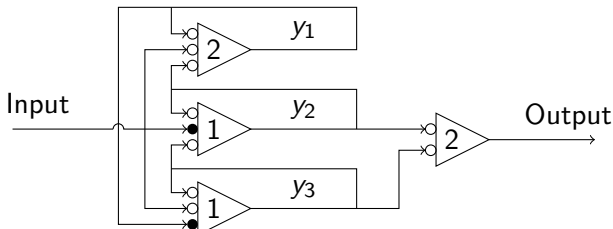
Warren Sturgis McCullough



Walter Pitts

- Came up with a mathematical model of a neuron
- Motivation to compare the computational power of networks of neurons with Turing Machines

## Simple Neural Net



A Simple Neural Net

- Neurons have excitatory (circles) or inhibitory (dots) synapses
- Neuron produces 1 if the number of excitatory synapses with 1-input exceeds the number of inhibitory synapses with 1-input by at least the threshold of the neuron (number inside triangle)

# Finite State Automata

- Finite automata model introduced by Huffman (1954), Moore (1956) and Mealy (1955) to model sequential circuits
- **Kleene, 1956**: Neural Nets of McCullough-Pitts the same as Finite Automata

# Nondeterminism

Michael Rabin and Dana Scott (1959)



Michael Rabin



Dana Scott

- Introduced the notion of Nondeterministic Finite Automata
- Understanding the power of nondeterministic computation has remained a fundamental problem since then

# Three Tales of Computation

- What is the nature of infinity?
- What enables humans to communicate through language?
- How does the human brain work?

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Common to these diverse threads of science is computation.  
They laid the foundations of a theory that we will explore.

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    - Approximate solutions, probabilistic solutions, partial solutions, . . .

# Course Overview

The three main computational models/problem classes in the course

Computational Model	Applications
Finite State Machines/ Regular Expressions	text processing, lexical analysis, protocol verification
Pushdown Automata/ Context-free Grammars	compiler parsing, software modeling, natural language processing
Turing machines	undecidability, computational complexity, cryptography