Pick the correct alternative from among the choices (A), (B), and (C) provided for each question below.

1. Let A and B be disjoint, recursively enumerable languages. Further let $\overline{A \cup B}$ also be recursively enumerable. What can you say about A and B?

   (A) It is possible that neither A nor B is decidable.

   (B) At least one among A and B is decidable.

   (C) Both A and B are decidable.

   The correct answer is (C). The decision procedure for A (and B) is as follows. Run the TMs for A, B and $\overline{A \cup B}$ simultaneously on the input. One of these is guaranteed to accept. Accept only if TM for A (B) accepts.

2. Just as we encoded Turing Machines as binary strings, we can also encode DFAs as binary strings. Let $\langle M \rangle_{\text{DFA}}$ be the binary string encoding of DFA M. Consider the following language $L_d^{\text{DFA}} = \{\langle M \rangle_{\text{DFA}} \mid \langle M \rangle_{\text{DFA}} \notin L(M)\}$. What can we say about L_d^{DFA}?

   (A) L_d^{DFA} is regular.

   (B) L_d^{DFA} is not regular but it is decidable.

   (C) L_d^{DFA} is not recursively enumerable.

   The correct answer is (B). Non-regularity follows for the same reasons that the diagonal language is not r.e.

3. Just as we encoded Turing Machines as binary strings, we can also encode PDAs as binary strings. Let $\langle M \rangle_{\text{PDA}}$ be the binary string encoding of PDA M. Consider the following language $L_d^{\text{PDA}} = \{\langle M \rangle_{\text{PDA}} \mid \langle M \rangle_{\text{PDA}} \notin L(M)\}$. What can we say about L_d^{PDA}?

   (A) L_d^{PDA} is decidable

   (B) L_d^{PDA} is not decidable but it is recursively enumerable.

   (C) L_d^{PDA} is not recursively enumerable.

   The correct answer is (A). Decidability follows from the fact that PDA membership is decidable (CYK algorithm).

4. Let L be decidable. Which of the following is true about L?

   (A) If $L' \subseteq L$ then L' is decidable.

   (B) If $L \subseteq L'$ then L' is decidable.

   (C) $L \leq_m \{0^n 1^n \mid n \geq 0\}$

   The correct answer is (C).

5. Let A and B be any languages such that $A \leq_m B$. Under what conditions is it the case that $\overline{A} \leq_m \overline{B}$?

   (A) Only when both A and B are decidable.

   (B) Only when both A and B are recursively enumerable.

   (C) Always.

   The correct answer is (C). Let f be a reduction from A to B. Then f is a computable and $x \in A$ iff $f(x) \in B$ which is the same as $x \notin A$ iff $f(x) \notin B$ which is the same as $x \in \overline{A}$ iff $f(x) \in \overline{B}$. Thus f is a reduction from $\overline{A}$ to $\overline{B}$.

6. Recall that $A_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$. Suppose $A_{\text{TM}} \leq_m L$ and $A_{\text{TM}} \leq_m \overline{L}$. What can we say about L?

   (A) L is not recursively enumerable.

   (B) L is recursively enumerable.

   (C) L is decidable.

   The correct answer is (A). L is not decidable because A_{TM} is not decidable and $A_{\text{TM}} \leq_m L$. L is not r.e. because $A_{\text{TM}} \leq_m \overline{L}$ means that $\overline{A_{\text{TM}}} \leq_m L$ (see previous part) and we know that $\overline{A_{\text{TM}}}$ is not r.e.

7. Which of the following **is** a property of recursively enumerable languages?

   (A) $\{\langle M\rangle \mid M \text{ accepts 312929 strings}\}$

   (B) $\{\langle M\rangle \mid M \text{ has 312929 states}\}$

   (C) $\{\langle M\rangle \mid M \text{ has 312929 symbols in tape alphabet}\}$

   The correct answer is (A).

8. Which of the following **is not** a property of recursively enumerable languages?

   (A) $\{\langle M\rangle \mid M \text{ accepts 0011}\}$

   (B) $\{\langle M\rangle \mid L(M) \text{ is accepted by a TM with even number of states}\}$

   (C) $\{\langle M\rangle \mid M \text{ uses no more than 32 tape cells}\}$

   The correct answer is (C).

9. Let $L = \{\langle M\rangle \mid M \text{ is a TM that accepts at least 312929 strings}\}$.

   (A) L is decidable.

   (B) L is not decidable but is recursively enumerable.

   (C) L is not recursively enumerable.

   The correct answer is (B). It is not decidable because of Rice's theorem, and it is r.e. because you can just guess 312929 strings and confirm that they all belong to the language.

10. Let $L = \{\langle M\rangle \mid M \text{ is a TM that accepts at most 312929 strings}\}$.

    (A) L is decidable.

    (B) L is not decidable but is recursively enumerable.

    (C) L is not recursively enumerable.

    The correct answer is (C). L is the complement of the language in the previous part. Since that is r.e. but not recursive, it follows that L is not r.e.