CSE 401/CS 450/MATH 450/ECE 491: Final Guide, Fall 2007

Final Exam, December 10th, 2007, 8 – 11 AM, 13## DCL

General Advice: The exam will be multiple choice with a difficulty level similar to that of the quizzes, midterm, and end of chapter review questions. A specific list of the scope of topics on the exam is included at below. In particular, the exam will be restricted to the sections listed. For your convenience the key topics, formula, and problems from each chapter are listed as well. Based on previous experience, a good strategy for studying for the final is as follows: (1) Study all prior quiz and midterm questions. (2) Understand all of the concepts given in the list at the end of this document. (3) Work through the examples in the book. (4) Be able to actually work simple problems (e.g., 2x2, 3x3, 3x2, triangular, diagonal, etc...). (5) For each chapter: existence/uniqueness, conditioning, key methods, relative (and absolute) complexity and stability of the key methods. (6) Look over the review questions at the end of each chapter.

[1] Sections 1.1 to 1.3.9

- Error, Accuracy, Stability:
  Absolute Error, Relative Error
  Data Error, Computational Error
  Truncation Error , Rounding Error
  Forward Error, Backward Error
  Well Conditioned, Stable, Well Posed
- Floating-Point Arithmetic:
  Underflow, Overflow, Machine Precision
  Subnormals, Gradual Underflow
  Cancellation
- Important Formula:
  Absolute Error, Relative Error
  Forward Error, Backward Error
  Condition Number
  Machine precision definitions on page 5
- Know how to compute:
  Forward/Backward Error
  Absolute/Relative Error
  fl(x+y), fl(x-y), fl(x*y), fl(x/y)
  E.g., when does 1 + a = 1 ?

[2] Sections 2.1 to 2.5.3

- Existence and Uniqueness:
  Nonsingular, Singular
- Conditioning:
  Vector and Matrix Norms
  Condition Number
  Error bounds, residual
- Solving:
  Upper and Lower Triangular
  Forward and Back Substitution
  Elementary Elimination Matrices
  Partial or Row Pivoting, Complete Pivoting
  LU Factorization, Gaussian Elimination
  Complexity of LU vs Inversion
  Complexity of solving modified systems
  Symmetric, Positive Definite, Banded, Sparse
  Cholesky
  Complexity for banded systems
- Important Formula:
  1-norm, 2-norm, inf-norm, p-norm for vectors
  1-norm and inf-norm for matrices
  Condition Number
  Properties of vector norms
  Properties of matrix norms
  Properties of the Condition Number
  Error bounds using condition number
  Elementary Elimination Matrices
- Know how to compute:
  Solution to triangular systems
  LU Factorizations (2x2 or 3x3 matrices)
  Norms and condition numbers of diagonal matrices
  Solve small linear systems of equations

[3] Sections 3.1 to 3.7 (exclude 3.4.2)

- Existence and Uniqueness:
  Span(A), Rank(A)
  Normal Equations
  Orthogonality of Span(A) with Residual
- Conditioning:
  Pseudoinverse and condition number
  Conditioning of the normal equations
- Solving:
  Normal Equations
  Triangular systems
  QR Factorization
  Householder, Givens
  Gram-Schmidt theory (not actual formula)
  Singular Value Decomposition
  2-norm and 2-norm condition number with SVD
  Pseudoinverse with SVD
  Comparison of methods (work vs stability)
- Important Formula:
  Normal Equations, Pseudoinverse, condition number
  Error bounds involving condition number
  Householder transformation
  Givens Rotation
  2-norm (condition number) using SVD
- Know how to compute:
  Setup and solution small least-squares problem (3x2)
  Normal equations
  Householder and Givens transformations
  Norms, condition numbers given the SVD

[4] Sections 4.1 to 4.7 (exclude 4.2.5, 4.5.8-4.5.11, 4.6)

- Existence and Uniqueness:

Characteristic Polynomial using determinant
Companion Matrix
Algebraic and Geometric multiplicity
Invariant subspace using eigenvectors
Diagonal, Tridiagonal, Triangular, Hessenberg
Orthogonal, Unitary, Symmetric, Hermitian, Normal
Which matrices are diagonalizable?
Which matrices have real eigenvalues?
Diagonalizable using general similarity transforms?
Diagonalizable using orthogonal/unitary transforms?

- Conditioning:
  Bound involving cond of matrix of eigenvectors
  Bound involving angle between right/left eigenvectors
- Solving:
  Shift, Inversion, Powers, Polynomials of A
  Similarity Transformations
  Unitary and Orthogonal Similarity Transforms
  Eigenvalues of diagonal, triangular, block triangular
  Defective matrices are not diagonalizable
  Normalized power iteration with/without shifts
  Normalized inverse iteration with/without shifts
  Rayleigh quotient
  Rayleigh quotient iteration
  Deflation
  Jordon, Schur, and Real Schur forms
  QR iteration with/without shifts
  Preliminary reduction to Hessenberg or Tridiagonal
  Properties of Lanczos and Arnoldi
  Comparison of methods (stability vs work)
- Important Formula:
  Characteristic Polynomial
  Bounds on p. 167 and 168
  Eigenvalues after shift, inverse, powers, polynomial
  Rayleigh quotient
- Know how to compute:
  Eigenvalues of a small matrix (2x2)
  Eigenvalues of a diagonal or triangular matrix
  Eigenvectors of a small matrix (2x2)
  1-step of the normalized power iteration
  1-step of inverse iteration (for triangular matrix)
  Rayleigh quotient

[5] Sections 5.1 to 5.6.4

- Existence and Uniqueness:
  Scalar and Systems case
  Single and multiple roots
- Conditioning:
  Derivative and Jacobian
  Multiple roots
  Condition number
- Rates of convergence:
  Linear, superlinear, quadratic
  Number of correct digits per iteration
- Solving Scalar Problems:
  Bisection
  Stability, rate of convergence of bisection
  Fixed point iterations
  Conditions for convergence of fixed point iterations
  Rate of convergence of fixed point iterations
  Newton's method
  Secant method
  Inverse interpolation (gen. concept/props.)
  Linear fractional interpolation (gen. concept/props.)
  Safeguard methods
  Zeros of polynomials by Newton's method
  Zeros of polynomials by companion matrix
  General comparison of methods for scalar problems
- Solving Systems of Nonlinear Equations:
  Newton's Method
  Convergence of a general fixed-point scheme
  Features of Broyden's method
  Trust Region and Damped Newton
- Important Formula:
  Condition number for root finding
  Convergence rate
  Convergence of (scalar/systems) fixed point iteration
- Know how to compute
  Jacobian of a system of equations
  Condition number for root finding
  1 or 2 steps of bisection
  1 step of a fixed point iteration
  1 step of Newton's method
  1 step of Secant method
  1 step of Newton's method for systems

[6] Sections 6.1 to 6.7.2 (excluding 6.5.7, 6.6.2, 6.7.1)

- Existence and Uniqueness:
  Global vs Local optimization
  Coercive
  Closed and Bounded
  Convex Function
  Convex Set
  First Order necessary condition
  Second Order sufficient (but not necessary) condition
  First Order necessary condition for constrained systems
  (Excluded: second order condition for constrained systems)
- Conditioning:
  Eigenvalues of Hessian
  (Excluded: sensitivity of constrained systems)
- Solving:
  Golden Section Search
  Successive Parabolic Interpolation (general idea)
  Newton's Method
  Safe Guarding
  Direct Search
  Steepest Descent
  Trust Region and Trust Radius
  BFGS (general features)
  Conjugate gradient (general features)
  Gauss-Newton (general features)
  Penalty and Barrier methods (general idea)
- Important Formula:
  Conditioning for unconstrained optimization
- Know how to compute:
  One step of Newton's method for optimization
  One step of Steepest descent for a quadratic function
  Hessian for an objective function

Eigenvalues of a Hessian at a critical point.

[7] Sections 7.1 to 7.4.3

- Existence, Uniqueness, and Conditioning:
  Depends on the form of the interpolating function
- Solving:
  Monomial vs. Newton vs. Lagrange
  Chebyshev Points
  Oscillation problem for equally spaced points
  Hermite Cubic
  Cubic Spline
  B-Splines (basic idea)
- Important Formula:
  Bound for error interpolating a smooth function (p 324)
- Know how to compute:
  Poly. interpolant in Monomial/Newton/Lagrange basis
  Incremental Newton
  Number of free parameters for spline interpolant
  Number of free parameters for Hermite interpolant
  "Extra conditions" for Spline and Hermite interpolants

[8] Sections 8.1 to 8.7 (excluding 8.3.2, 8.4.2 to 8.5, 8.6.2)

- Existence and Uniqueness:
  Bounded functions with finite number of discontinuities
  Sufficiently smooth functions are differentiable
- Conditioning:
  Integration is well conditioned for continuous functions
  Differentiation is ill conditioned for noisy data
- Solving:
  Newton-Cotes
  Trapezoid and Midpoint rule
  Gaussian Quadrature
  Progressive Quadrature
  Composite Quadrature
  Composite Midpoint and Trapezoid rule
  Adaptive quadrature (general idea)
  Forward, Backward, Centered difference approximations.
  Richardson Extrapolation
- Important Formula:
  General form of bounds for error for Midpoint/Trapezoid
- Know how to compute:
  Composite Trapezoid/Midpoint given tabular f(x) values
  Richardson extrapolated value for a method of order p

[9] Sections 9.1 to 9.3.8 (excluding 9.3.5)

- Existence and Uniqueness:
  Conditioning
  Linear stability conditions
  Stable vs Asymptotically Stable vs Unstable
  Non-linear stability conditions
- Solving:
  Local Error vs Global Error
  Converting a kth-order problem into first order form
  Forward Euler, Backward Euler, and Trapezoid
  Stability for Euler, Backward Euler, and Trapezoid
  Stiffness
  Necessity of Implicit method for Stiff problems
  Runge-Kutta Methods (general features)
  Multistep methods (general features)
  Predictor Corrector Methods (general features)
- Important Formula:
  Stability for Forward/Backward Euler and Trapezoid
  Stability for linear and nonlinear problems
- Know how to compute:
  One step of forward Euler (scalar and systems)
  One step of backward Euler (scalar and systems)
  One step of trapezoid rule (scalar)
  Stability for a linear problem
  Stability for a method applied to a linear problem

[10] Sections 10.1 to 10.6

- Existence and Uniqueness and Conditioning
  Modes are coupled to boundary conditions (gen. concept)
- Definitions:
  Separated Boundary Conditions
  Linear Boundary Value Problem
- Solving:
  Steps for the Shooting Method
  Steps for the Finite Difference Method
  Steps for the Collocation Method
- Know how to compute:
  Three-point finite difference solution
  Quadratic collocation solution

[11] Sections 11.1 to 11.3.2

- Existence and Uniqueness and Conditioning:
  Know it's complicated
- Types of PDEs:
  Advection Equation
  Heat Equation
  Wave Equation
  Laplace Equation
  Dirichlet vs Neumann boundary conditions
- Solving:
  Characteristics of the advection equation
  Well posed boundary conditions and characteristics
  Domain of dependence and characteristics
  Features/Examples of Hyperbolic PDEs
  Features/Examples of Parabolic PDEs
  Features/Examples of Elliptic PDEs
  Semidiscrete Methods and Method of Lines
  Fully discrete methods (e.g. Crank-Nicolson)
- Know how to compute:
  Identify PDE type given one of the four simple examples
  Identify PDE type given its features
  Characteristics of the advection equation
  Simple Solve for 2D Fully Discrete (e.g. Ex 11.2)