CS 273: Intro to Theory of Computation, Fall 2007
Head-banging 6 (17-19 Oct)

1. **Pumping Lemma for regular languages** Let $\Sigma = \{0,1\}$. For each of the following languages, find a string $s_p = xyz$ for each potential pumping length, and give the number of times, i, that the string x needs to be pumped to get something not in the language.
(a) $L = \{(01)^n(10)^n \mid n \in \mathbb{N}_0\}$
(b) $L = \{0^{(2^k)} \mid k \in \mathbb{N}_0\}$
(c) $L = \{x_1\#x_2\#\ldots\#x_n\#y \mid x_1,\ldots,x_n \in \Sigma^*, |x_1| \geq |y| > 0, y = x_k \text{ for some } k\}$

Solution: (a) $(01)^p(10)^p, i = 2$. This will shift the position of the pair of adjacent ones, or create another pair of adjacent ones.
(b) $2^{2^p}, i = 0$. $2^{2^p} - x$, where $1 \leq x \leq p$ is not a power of two for any $p \geq 1$.
(c) $0^p\#\#\ldots\#0^p, i = 0$. After pumping down, $y \neq x_i$ for any i.

3. **Proving that languages are not context-free**
(a) Using the pumping lemma for context free languages, prove that the following language is not context free:

$$\{0^{2^k} \mid k \in \mathbb{N}_0\}$$

(b) Use part a and closure properties to show that $\{1^n 0^{2^n} 1^n 0^k \mid k, n \in \mathbb{N}_0\}$ is not context free

Solution: (a) Assume that $L = \{0^{2^k} \mid k \in \mathbb{N}_0\}$ is context free, and hence satisfies the pumping lemma. Then there is a p such that $0^{2^{2^p}} = uvxyz$ such that $uv^0xy^0z = 0^{2^{2^p}-|vy|} \in L$, and $1 \leq |vy| \leq p$. But again, $2^{2^p} - x$ is not a power of two for any $p \geq 1$, $x \leq p$. Thus $0^{2^{2^p}-|vy|} \notin L$, and so L is not context-free.

(b) Intersect with the regular language $1^*0^*1^*$, and use a homomorphism to map 1 to ϵ. This yields $\{0^{2^n} \mid n \in \mathbb{N}_0\}$, which is not context free.