

CS 273: Intro to Theory of Computation, Fall 2007

Head-banging 11 (28 - 30 Nov)

1. Quick warm up

Are the following languages decidable or undecidable?

- $\{ \langle M \rangle \mid M \text{ is a TM and } \epsilon \in L(M) \}$
- $\{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is finite} \}$
- $\{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is regular} \}$
- $\{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is context-free} \}$
- $\{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is the set of all palindromes} \}$
- $\{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is decidable} \}$

If you have not notice already what this problem is really about please ask one of the TA's.

2. A_{TM} a.k.a. universal language

This is a quick review, and you should know this without looking at your book.

- Define A_{TM} .
- Briefly explain why $\overline{A_{TM}}$ is undecidable. In fact, $\overline{A_{TM}}$ is not Turing-recognizable. Briefly explain why this is also the case.

3. Reduction

Prove that the language

$$\{ \langle M, w \rangle \mid M \text{ is a TM with input string } w \text{ does not halt} \}$$

is undecidable. (Hint: use $\overline{A_{TM}}$)

4. More Reduction

Let L_{TAGGER} be the language

$$\{ \langle M \rangle \mid \text{On empty input, } M \text{ changes the contexts of the tape cell on every move} \}$$

That is, each time M makes a transition, it replaces the symbol in the current tape cell with a different symbol.

If the TM changes some tape symbol from (say) a to b , goes off and does other things, and then comes back to the same tape position, it must change the b to some other symbol. It might change the b to some third character c , or it could change it back to the original character a .

Show that L_{TAGGER} is undecidable, using a reduction from A_{TM} .