

Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

First Class Types

- A type is *first class* if it can be
 - **Passed** as an argument
 - **Assigned** as a value
 - **Returned** as a result
- Examples:
 - C: scalars, pointers, structures
 - C++: same as C, plus classes
 - Scheme, LISP: scalars, lists (s-expressions), functions
 - ML: same as Scheme, plus user defined data types

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First Class Types

- The kind of data that can be manipulated well in a language largely determines for which applications the language is well suited
- The ability to treat functions as data is one of the strengths of applicative programming languages

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Higher Order Functions

- A function is *higher-order* if it takes a function as an argument or returns one as a result
- Example:

```
# let compose f g = fun x -> f (g x);;  
val compose : ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b = <fun>
```

- The type $(a \rightarrow b) \rightarrow (c \rightarrow a) \rightarrow c \rightarrow b$ is a higher order type because of $(a \rightarrow b)$ and $(c \rightarrow a)$ and $\rightarrow c \rightarrow b$

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Higher Order Functions

- What are the types of the following functions:

```
# plus_two;;  
- : int -> int = <fun>  
# compose plus_two plus_two;;  
?  
# compose plus_two;;  
?
```

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Higher Order Functions

- What are the types of the following functions:

```
# plus_two;;  
- : int -> int = <fun>  
# compose plus_two plus_two;;  
- : int -> int = <fun>  
# compose plus_two;;  
- : ('a -> int) -> 'a -> int = <fun>
```

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Higher Order Functions

- What do the following functions do?

```
# plus_two;;  
- : int -> int = <fun>  
# compose plus_two plus_two;;  
- : int -> int = <fun>  
# compose plus_two;;  
- : ('_a -> int) -> '_a -> int = <fun>
```

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Thrice

- Recall:

```
# let thrice f x = f (f (f x));;  
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
```

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Thrice

- Recall:

```
# let thrice f x = f (f (f x));;  
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
```

- How do you write thrice with compose?

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Thrice

- Recall:

```
# let thrice f x = f (f (f x));;  
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
```

- How do you write thrice with compose?

```
# let thrice f = compose f (compose f f);;  
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
```

- Is this the only way?

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Reversing Arguments

```
# let flip f a b = f b a;;  
val flip : ('a -> 'b -> 'c) -> 'b -> 'a -> 'c = <fun>  
# map ((-) 1) [5;6;7];;  
- : int list = [-4; -5; -6]  
# map (flip (-) 1) [5;6;7];;  
- : int list = [4; 5; 6]  
# let (-) = flip (-);;  
val (-) : int -> int -> int = <fun>  
# 2 - 5;;  
- : int = 3
```

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Partial Application

```
# (+);;  
- : int -> int -> int = <fun>  
# (+) 2 3;;  
- : int = 5  
# let plus_two = (+) 2;;  
val plus_two : int -> int = <fun>  
# plus_two 7;;  
- : int = 9
```

- Partial application also called *sectioning*

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Lambda Lifting

- You must remember the rules for evaluation when you use partial application

```
# let add_two = (+) (print_string "test\n"; 2);;
test
val add_two : int -> int = <fun>
# let add2 = (* lambda lifted *)
  fun x -> (+) (print_string "test\n"; 2) x;;
val add2 : int -> int = <fun>
```

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Lambda Lifting

```
# thrice add_two 5;;
- : int = 11
# thrice add2 5;;
test
test
test
- : int = 11
```

- Lambda lifting delayed the evaluation of the argument to (+) until the second argument was supplied

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Lambda Lifting and “Unknown Types”

- Recall `compose plus_two`:

```
# let f1 = compose plus_two;;
val f1 : ('a -> int) -> 'a -> int = <fun>
```

- Compare to lambda lifted version:

```
# let f2 = fun g -> compose plus_two g;;
val f2 : ('a -> int) -> 'a -> int = <fun>
```

- What is the difference?

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Lambda Lifting and “Unknown Types”

- 'a can only be instantiated once for an expression

```
# f1 plus_two;;
- : int -> int = <fun>
# f1 List.length;;
Characters 3-14:
  f1 List.length;;
  ^^^^^^^^^^^^^
```

This expression has type 'a list -> int but is here used with type int -> int

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Lambda Lifting and “Unknown Types”

- 'a can be repeatedly instantiated

```
# f2 plus_two;;
- : int -> int = <fun>
# f2 List.length;;
- : 'a list -> int = <fun>
```

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Curried vs Uncurried

```
val add_three : int -> int -> int -> int = <fun>
How does it differ from
# let add_triple (u,v,w) = u + v + w;;
val add_triple : int * int * int -> int = <fun>
```

`add_three` is *curried*;
`add_triple` is *uncurried*

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curry and uncurry

```
# let curry f x y = f (x,y);;
val curry : ('a * 'b -> 'c) -> 'a -> 'b -> 'c =
  <fun>
# let uncurry f (x,y) = f x y;;
val uncurry : ('a -> 'b -> 'c) -> 'a * 'b -> 'c =
  <fun>
```

- Remember these!

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curry and uncurry

```
# (+);;
- : int -> int -> int = <fun>
# let plus = uncurry (+);;
val plus : int * int -> int = <fun>
# plus (3,4);;
- : int = 7
# curry plus 3 4;;
- : int = 7
```

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Folding Functions over Lists

How are the following functions similar?

```
# let rec sumlist list = match list with
  [] -> 0 | x::xs -> x + sumlist xs;;
val sumlist : int list -> int = <fun>
# sumlist [2;3;4];;
- : int = 9
# let rec prodlist list = match list with
  [] -> 1 | x::xs -> x * prodlist xs;;
val prodlist : int list -> int = <fun>
# prodlist [2;3;4];;
- : int = 24
```

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Folding

```
# let rec fold_left f a list = match list
  with [] -> a | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a = <fun>
fold_left f a [x1; x2; ...; xn] = f(...(f (f a x1) x2)...)xn
# let rec fold_right f list b = match list
  with [] -> b | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b = <fun>
fold_right f [x1; x2; ...; xn] b = f x1(f x2(... (f xn b)...))
```

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Folding

```
# let sumlist list = fold_right (+) list 0;;
val sumlist : int list -> int = <fun>
# sumlist [2;3;4];;
- : int = 9
# let prodlist list = fold_right ( * ) list 1;;
val prodlist : int list -> int = <fun>
# prodlist [2;3;4];;
- : int = 24
```

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Folding

- Can replace recursion by `fold_right` in any primitive recursive definition
 - Primitive recursive means it only recurses on immediate subcomponents of recursive data structure
- Can replace recursion by `fold_left` in any tail recursive definition

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Encoding Recursion with Fold

```
# let rec append list1 list2 = match list1 with
  [] -> list2 | x::xs -> x :: append xs list2;;
val append : 'a list -> 'a list -> 'a list = <fun>
# let rec append list1 list2 =
  fold_right (fun x y -> x :: y) list1 list2;;
val append : 'a list -> 'a list -> 'a list = <fun>
# append [1;2;3] [4;5;6];;
- : int list = [1; 2; 3; 4; 5; 6]
```

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Base Case

Operation

Recursive Call

Combining Lists of Functions

```
# let rec complist flist = match flist with
  [] -> (fun x -> x)
  | f::fs -> compose f (complist fs);;
val complist : ('a -> 'a) list -> 'a -> 'a = <fun>
# complist [( - ) 1; ( * ) 3; plus_two] ;;
- : int -> int = <fun>
# complist [( - ) 1; ( * ) 3; plus_two] 5;;
- : int = -20
# complist [( - ) 1; ( * ) 3; plus_two] 5;;
- : int = -20
```

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Why isn't type more general, like compose?

Can you write this with fold_right?

Repeating n Times

```
# let rec repeat n f x =
  match n with 0 -> x | _ -> f (repeat (n - 1) f x);;
val repeat : int -> ('a -> 'a) -> 'a -> 'a = <fun>
# repeat 8 (fun x -> x * 2) 1;;
- : int = 256
# let rec iter n f x =
  match n with 0 -> x | _ -> iter (n - 1) f (f x);;
val iter : int -> ('a -> 'a) -> 'a -> 'a = <fun>
# iter 8 (fun x -> x * 2) 1;;
- : int = 256
```

- Which is more efficient?

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Mapping

- What do these functions have in common?

```
# let rec inclist list = match list with [] -> []
  | x :: xs -> (1 + x) :: inclist xs;;
val inclist : int list -> int list = <fun>
# inclist [2;3;4];;
- : int list = [3; 4; 5]
# let rec doublelist list = match list with [] -> []
  | x :: xs -> (2 * x) :: doublelist xs;;
val doublelist : int list -> int list = <fun>
# doublelist [2;3;4];;
- : int list = [4; 6; 8]
```

Recall Map

```
# let rec map f list =
  match list
  with [] -> []
  | (h::t) -> (f h) :: (map f t);;
val map : ('a -> 'b) -> 'a list -> 'b list =
  <fun>
# map plus_two fib5;;
- : int list = [10; 7; 5; 4; 3; 3]
```

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Mapping

```
# let inclist = map ((+) 1);;
val inclist : int list -> int list = <fun>
# inclist [2;3;4];;
- : int list = [3; 4; 5]
# let doublelist = map ((* ) 2);;
val doublelist : int list -> int list = <fun>
# doublelist [2;3;4];;
- : int list = [4; 6; 8]
```

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Map from Fold

```
# let map f list =  
  fold_right (fun x y -> f x :: y) list [ ];;  
val map : ('a -> 'b) -> 'a list -> 'b list = <fun>  
# map ((+)1) [1;2;3];;  
- : int list = [2; 3; 4]
```

- Can you write `fold_right` (or `fold_left`) with just `map`? How, or why not?

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Related Function: Zip

```
# let rec zip list1 list2 =  
  match (list1,list2) with ([ ], _) -> []  
  | (_, [ ]) -> []  
  | (x::xs, y::ys) -> (x,y)::zip xs ys;;  
val zip : 'a list -> 'b list -> ('a * 'b) list =  
  <fun>  
# zip [1;2;3] [4;5;6];;  
- : (int * int) list = [(1, 4); (2, 5); (3, 6)]
```

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Zipwith

```
# let rec zipwith f list1 list2 =  
  match (list1,list2) with ([ ], _) -> []  
  | (_, [ ]) -> []  
  | (x::xs, y::ys) -> f x y ::zipwith f xs ys;;  
val zipwith : ('a -> 'b -> 'c) -> 'a list -> 'b list -> 'c  
  list = <fun>  
# zipwith (+) [1;2;3] [4;5;6];;  
- : int list = [5; 7; 9]  
# zipwith (fun x y -> (x,y)) [1;2;3] [4;5;6];;  
- : (int * int) list = [(1, 4); (2, 5); (3, 6)]
```

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Zip from Zipwith

- How do you write `zip` using `zipwith`, and no explicit recursion?

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Zip from Zipwith

- How do you write `zip` using `zipwith`, and no explicit recursion?

```
# let zip = zipwith (fun x y -> (x,y));;  
val zip : '_a list -> '_b list -> ('_a * '_b) list  
  = <fun>  
# zip [1;2;3] [4;5;6];;  
- : (int * int) list = [(1, 4); (2, 5); (3, 6)]
```

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Problems

- Write a function `flipuc` that flips the arguments to an uncurried function, using just `curry`, `flip` and `uncurry`
- Write a function that has type $('a \rightarrow 'b) \rightarrow 'a * 'c \rightarrow 'b$
- Use `fold_right` to write a function that takes a list and returns it.
- Use `fold_right` to write a function to remove all negative elements from a list

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Problem 1

- Write a function `flipuc` that flips the arguments to an uncurried function, using just `curry`, `flip` and `uncurry`

```
# let flipuc f = uncurry (flip (curry f));;
val flipuc : ('a * 'b -> 'c) -> 'b * 'a -> 'c = <fun>
# let cons (x,xs) = x::xs;;
val cons : 'a * 'a list -> 'a list = <fun>
# let snoc = flipuc cons;;
val snoc : 'a list * 'a -> 'a list = <fun>
# snoc(snoc ([1],2),3);;
- : int list = [3; 2; 1]
```

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Problem 2

- Write a function that has type
`('a -> 'b) -> 'a * 'c -> 'b`

```
# let app_fst f (a,b) = f a;;
val app_fst : ('a -> 'b) -> 'a * 'c -> 'b = <fun>
# app_fst ((+) 1) (3, 7);;
- : int = 4
# app_fst ((+) 1) (4, "hi");;
- : int = 5
```

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Problem 3

- Use `fold_right` to write a function that takes a list and returns it.

```
# let listId list =
  fold_right (fun x xs -> x::xs) list [];;
val listId : 'a list -> 'a list = <fun>
# listId [1;2;3];;
- : int list = [1; 2; 3]
```

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Problem 4

- Use `fold_right` to write a function to remove all negative elements from a list

```
# let gezero list =
  fold_right
  (fun x xs -> if x >= 0 then x::xs else xs)
  list [];;
val gezero : int list -> int list = <fun>
# gezero [1;0;3;-5;7;-2];;
- : int list = [1; 0; 3; 7]
```

- Related to filter function (in next homework)

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