

# Programming Languages and Compilers (CS 421)

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[http://www.cs.uiuc.edu/class  
/fa06/cs421/](http://www.cs.uiuc.edu/class/fa06/cs421/)

Based in part on slides by Mattox Beckman, as updated  
by Vikram Adve and Gul Agha

# Recursion Example

- Compute  $n^2$  recursively using:

$$n^2 = (2 * n - 1) + (n - 1)^2$$

```
# let rec nthsq n =      (* rec for recursion *)
  match n              (* pattern matching for cases *)
  with 0 -> 0          (* base case *)
  | n -> (2 * n - 1)   (* recursive case *)
      + nthsq (n - 1);; (* recursive call *)
val nthsq : int -> int = <fun>
# nthsq 3;;
- : int = 9
```

- Structure of recursion similar to inductive proof

# Recursion and Induction

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```
# let rec nthsq n = match n with 0 -> 0  
  | n -> (2 * n - 1) + nthsq (n - 1) ;;
```

- Base case is the last case; it stops the computation
- Recursive call must be to arguments that are somehow smaller - must progress to base case
- **if** or **match** must contain base case
- Failure of these may cause failure of termination

# Structural Recursion

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- Functions on recursive datatypes (eg lists) tend to be recursive
- Recursion over recursive datatypes generally by structural recursion
  - Recursive calls made to components of structure of the same recursive type
  - Base cases of recursive types stop the recursion of the function

# Structural Recursion : List Example

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```
# let rec length list = match list
  with [] -> 0 (* Nil case *)
  | x :: xs -> 1 + length xs;; (* Cons case *)
val length : 'a list -> int = <fun>
# length [5; 4; 3; 2];;
- : int = 4
```

- Nil case [] is base case
- Cons case recurses on component list xs

# Forward Recursion

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- In structural recursion, you split your input into components
- In forward recursion, you first call the function recursively on all the recursive components, and then build the final result from the partial results
- Wait until the whole structure has been traversed to start building the answer

# Forward Recursion: Examples

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```
# let rec double_up list =  
  match list  
  with [ ] -> [ ]  
       | (x :: xs) -> (x :: x :: double_up xs);;  
val double_up : 'a list -> 'a list = <fun>
```

```
# let rec poor_rev list =  
  match list  
  with [] -> []  
       | (x::xs) -> poor_rev xs @ [x];;  
val poor_rev : 'a list -> 'a list = <fun>
```

# Mapping Recursion

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- One common form of structural recursion applies a function to each element in the structure

```
# let rec doubleList list = match list  
  with [] -> []
```

```
   | x::xs -> 2 * x :: doubleList xs;;
```

```
val doubleList : int list -> int list = <fun>
```

```
# doubleList [2;3;4];;
```

```
- : int list = [4; 6; 8]
```

# Mapping Recursion

- Can use the higher-order recursive map function instead of direct recursion

```
# let doubleList list =  
  List.map (fun x -> 2 * x) list;;  
val doubleList : int list -> int list = <fun>  
# doubleList [2;3;4];;  
- : int list = [4; 6; 8]
```

- Same function, but no rec

# Folding Recursion

- Another common form “folds” an operation over the elements of the structure

```
# let rec multList list = match list
  with [ ] -> 1
       | x::xs -> x * multList xs;;
val multList : int list -> int = <fun>
# multList [2;4;6];;
- : int = 48
```

- Computes  $(2 * (4 * (6 * 1)))$

# Folding Recursion

- multList folds to the right
- Same as:

```
# let multList list =  
  List.fold_right  
    (fun x -> fun p -> x * p)  
    list 1;;  
  
val multList : int list -> int = <fun>  
  
# multList [2;4;6];;  
- : int = 48
```

# How long will it take?

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- Remember the big-O notation from CS 225 and CS 273
- Question: given input of size  $n$ , how long to generate output?
- Express output time in terms of input size, omit constants and take biggest power

# How long will it take?

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Common big-O times:

- Constant time  $O(1)$ 
  - input size doesn't matter
- Linear time  $O(n)$ 
  - double input  $\Rightarrow$  double time
- Quadratic time  $O(n^2)$ 
  - double input  $\Rightarrow$  quadruple time
- Exponential time  $O(2^n)$ 
  - increment input  $\Rightarrow$  double time

# Linear Time

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- Expect most list operations to take linear time  $O(n)$
- Each step of the recursion can be done in constant time
- Each step makes only one recursive call
- List example: multList, append
- Integer example: factorial

# Quadratic Time

- Each step of the recursion takes time proportional to input
- Each step of the recursion makes only one recursive call.
- List example:

```
# let rec poor_rev list = match list
  with [] -> []
       | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

# Exponential running time

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- Hideous running times on input of any size
- Each step of recursion takes constant time
- Each recursion makes two recursive calls
- Easy to write naïve code that is exponential for functions that can be linear

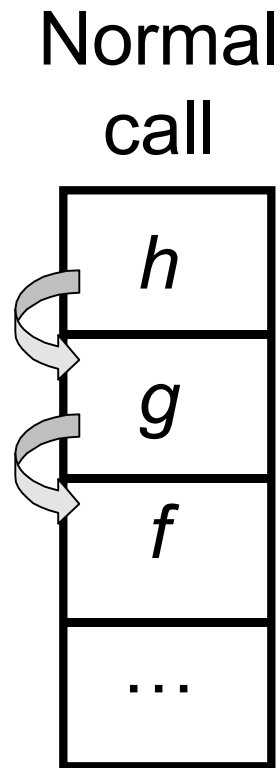
# Exponential running time

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```
# let rec naiveFib n = match n
  with 0 -> 0
      | 1 -> 1
      | _ -> naiveFib (n-1) + naiveFib (n-2);;
val naiveFib : int -> int = <fun>
```

# An Important Optimization

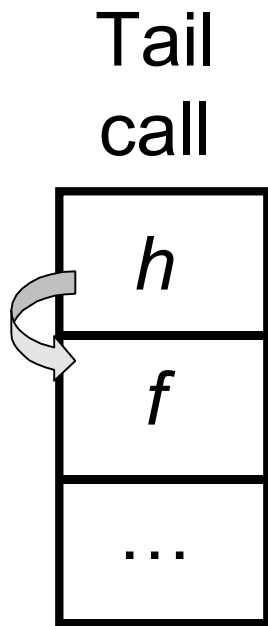
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- When a function call is made, the return address needs to be saved to the stack so we know to where to return when the call is finished
- What if *f* calls *g* and *g* calls *h*, but calling *h* is the last thing *g* does (a *tail call*)?

# An Important Optimization

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- When a function call is made, the return address needs to be saved to the stack so we know to where to return when the call is finished
- What if *f* calls *g* and *g* calls *h*, but calling *h* is the last thing *g* does (a *tail call*)?
- Then *h* can return directly to *f* instead of *g*

# Tail Recursion

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- A recursive program is tail recursive if all recursive calls are tail calls
- Tail recursive programs may be optimized to be implemented as loops, thus removing the function call overhead for the recursive calls
- Tail recursion generally requires extra “accumulator” arguments to pass partial results
  - May require an auxiliary function

# Tail Recursion - Example

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```
# let rec rev_aux list revlist =  
  match list with [ ] -> revlist  
  | x :: xs -> rev_aux xs (x::revlist);;  
val rev_aux : 'a list -> 'a list -> 'a list = <fun>
```

```
# let rev list = rev_aux list [ ];;  
val rev : 'a list -> 'a list = <fun>
```

- What is its running time?

# Comparison

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- `poor_rev [1,2,3] =`
- `(poor_rev [2,3]) @ [1] =`
- `((poor_rev [3]) @ [2]) @ [1] =`
- `((poor_rev [ ]) @ [3]) @ [2]) @ [1] =`
- `(( [ ] @ [3]) @ [2]) @ [1] =`
- `( [3] @ [2]) @ [1] =`
- `(3 :: ( [ ] @ [2])) @ [1] =`
- `[3,2] @ [1] =`
- `3 :: ([2] @ [1]) =`
- `3 :: (2 :: ([ ] @ [1])) = [3, 2, 1]`

# Comparison

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- $\text{rev } [1,2,3] =$
- $\text{rev\_aux } [1,2,3] [] =$
- $\text{rev\_aux } [2,3] [1] =$
- $\text{rev\_aux } [3] [2,1] =$
- $\text{rev\_aux } [] [3,2,1] = [3,2,1]$