# Decidable problems — CFLs, TMs

Note Title 10/31/2006

$$Mem_{DFA} = \{ \langle A, w \rangle \mid A \text{ is a DFA accepting } w \}$$

$$E_{DFA} = \{ \langle A \rangle \mid L(A) \neq \emptyset \}$$

Decidable. $\quad \|$

$Mem_{NFA}$

$E_{NFA}$

$$Mem_{CFG} = \{ \langle G, w \rangle \mid w \in L(G) \}$$

$$G: \quad A \to (A) \mid \varepsilon$$

$$\langle G, () \rangle \in Mem_{CFG}$$

$$\langle G, )( \rangle \notin Mem_{CFG}$$

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$G$ : Non-terminals, Terminals, Production Rules, Start Non-terminal.

$$A \longrightarrow aBbC$$

$$\$ A \to aBbC \$ \ldots \$$$

$$\mathrm{Mem}_{CFG} = \{ \langle G, w \rangle \mid L(G) \ni w \}$$

$$A \to BC \mid CD$$

$$A \to \cancel{AA} \mid A \to \varepsilon \mid a$$

- Generate parse trees of increasing size
- Check if any of them is a parse tree for $w$; if so accept (and halt)

Assume that $G$ is in CNF
(Chomsky Normal Form)

I. $A \to BC$

II. $A \to a$

I. Generates an extra nonterminal

II. ~~Clever~~ Converts a non-terminal to a terminal.

I is applicable only $|w| - 1 = n-1$ times

II is applicable only $n$ times.

Derivation for $w$ uses at most $2n+1$ rule applications

$Mem_{CFG} = \{ \langle G, w \rangle \mid w \in L(G) \}$

- Convert $G$ to CNF, $G'$
- Write down all derivations of length $2n+1$ where $n = |w|$
- Accept if any of them derive $w$
  - & reject if none of them do.

$$E_{CFG} = \{ \langle G \rangle \mid L(G) \neq \emptyset \}$$

$$G: \quad \boxed{\begin{aligned} &A \to BC \\ &B \to C \\ &C \to B \end{aligned}} \longrightarrow \boxed{B \to aB}$$

$$L(G) = \emptyset$$

$$L(G) = \{ w \mid A \to BC \to CC \to \cdots \to w \}$$

$$B \to aB \mid aC$$

$$C \to d$$

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$\mathcal{L}(G, NT)$ = Set of words generated by $G$ starting with <u>$NT$</u>

$A \longrightarrow BC$

$B \longrightarrow$

$A \longrightarrow$

$B \longrightarrow$

$C \longrightarrow$ abc

$\vdots$

$D \longrightarrow$

$A$ ✓

$B$ ✓

$C$ ✓

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$D$

$B \longrightarrow Cd$

$B \longrightarrow CC$

$A \rightarrow (A) \mid B$

$B \rightarrow [C]$

$C \rightarrow d$

A ✓
B ✓
C ✓

If $L(G) \neq \emptyset$,

$w \in L(G)$

parse tree for $w$

$B \to DE$


So initial nonterminal will get marked.

$$E_{CFG} = \{ \langle G \rangle \mid L(G) \neq \emptyset \}$$

is decidable.

$$F_{CFG} = \{ \langle G \rangle \mid L(G) = \emptyset \}$$

$$NonMemb = \{ \langle G, w \rangle \mid w \notin L(G) \}$$

is decidable.

$EQ_{CFG} = \{ \langle G, G' \rangle \mid L(G) = L(G') \}$

- Generate systematically all words in $\Sigma^*$
- Check if any of them belong to $L(G)$ and not in $L(G')$ OR not in $L(G)$ and in $L(G')$ then reject.

$NEQ_{CFG} = \{ \langle G, G' \rangle \mid L(G) \neq L(G') \}$

is recognizable.

EQ and NEQ are ("going to be") undecidable!

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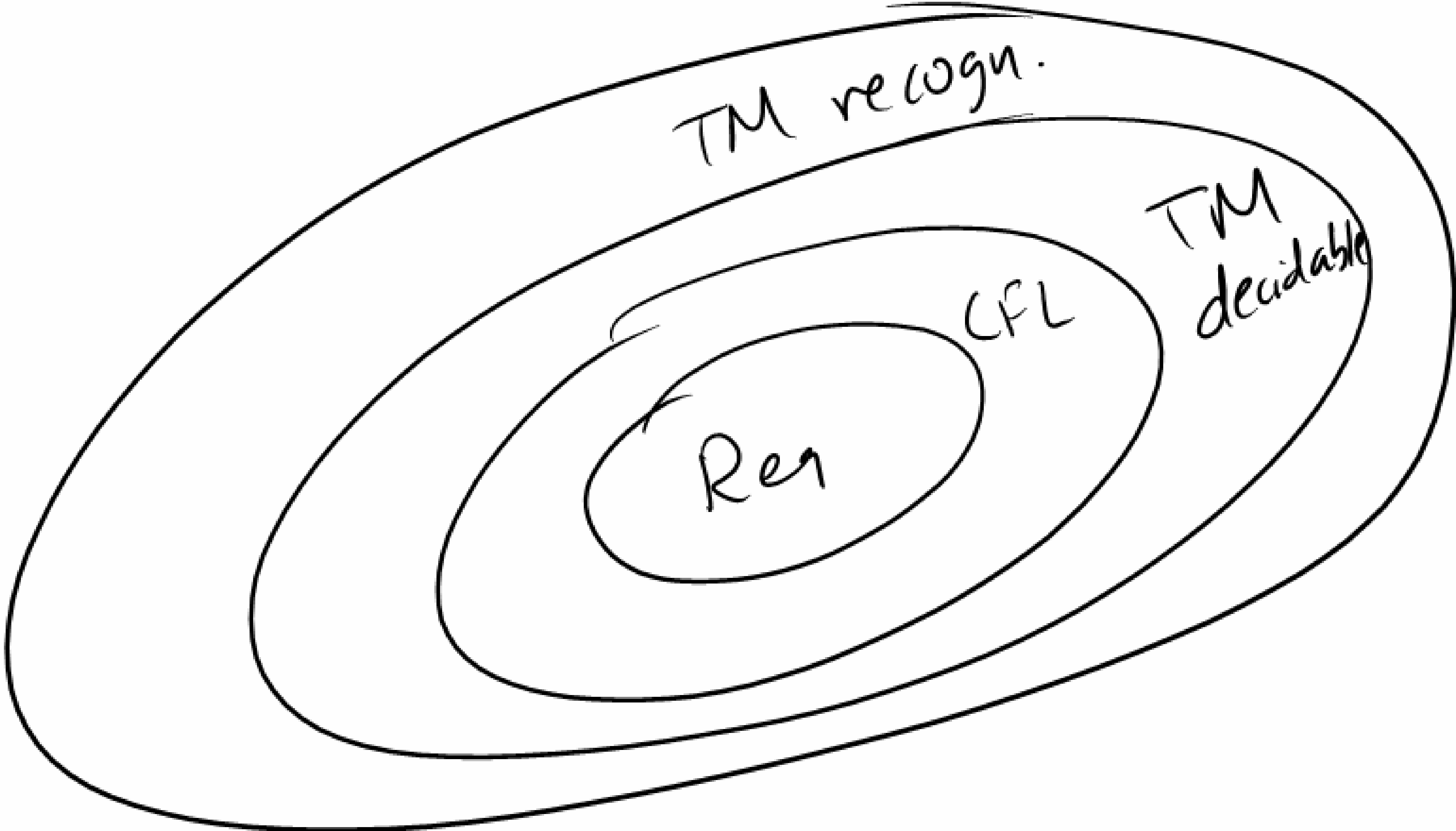
$EQ_{DFA}$ is decidable.

$A, A'$

$L(A) \subseteq L(A')$

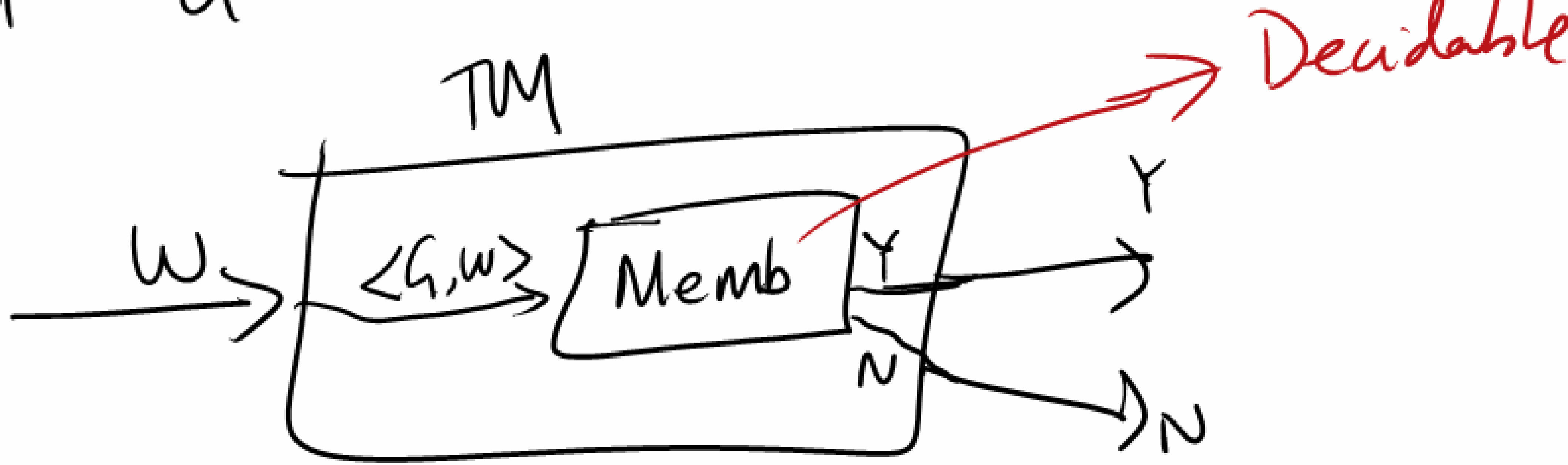
$L(A') \subseteq L(A)$

$L(A) \cap \overline{L(A')} = \emptyset$


TM recogn.
TM decidable
CFL
Reg

# Every CFL is decidable

Let $G$ be a CFG, $L(G) = L$

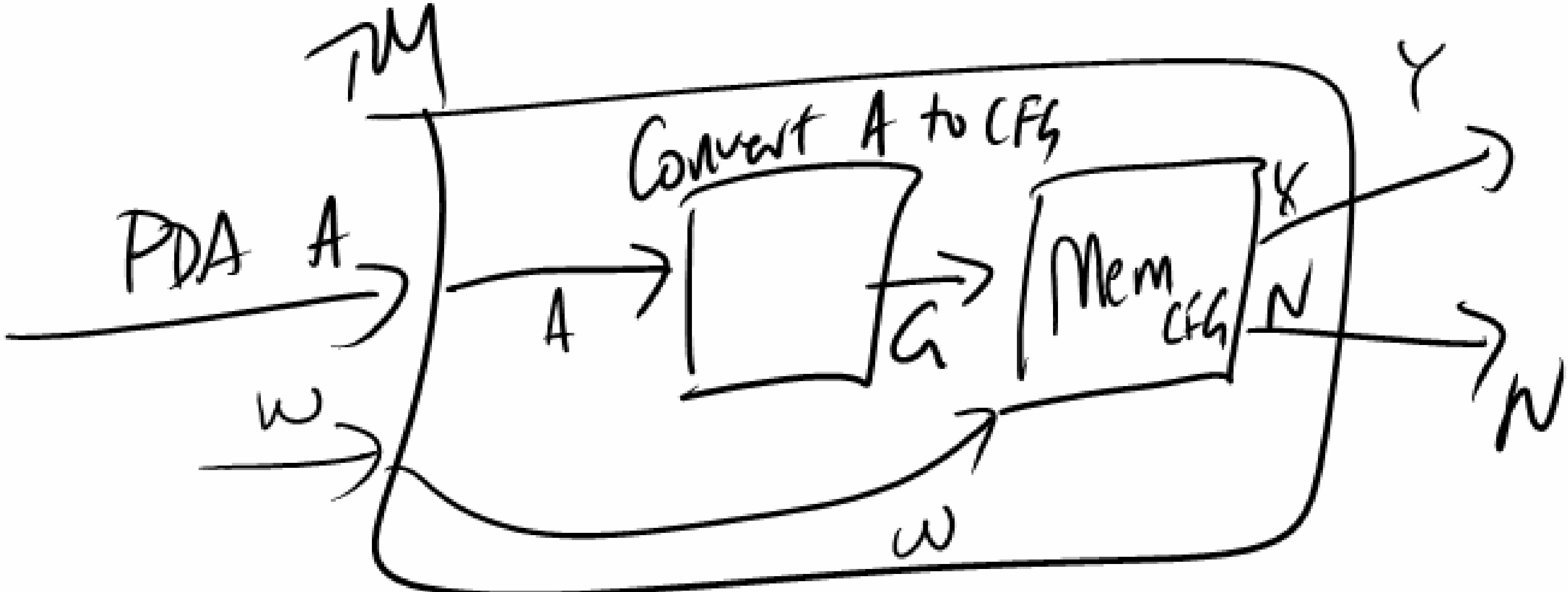



So $L$ is decidable.

$$Mem_{PDA} = \{ \langle A, w \rangle \mid A \text{ is a PDA and } w \in L(A) \}$$

$$PDA \equiv CFG$$

$$PDA \xrightarrow{\text{effective}} CFG$$



# Simulating Turing machines

$$TM = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

$$\langle Q \rangle \$ \langle \Sigma \rangle \$ \Gamma \$ \ldots$$

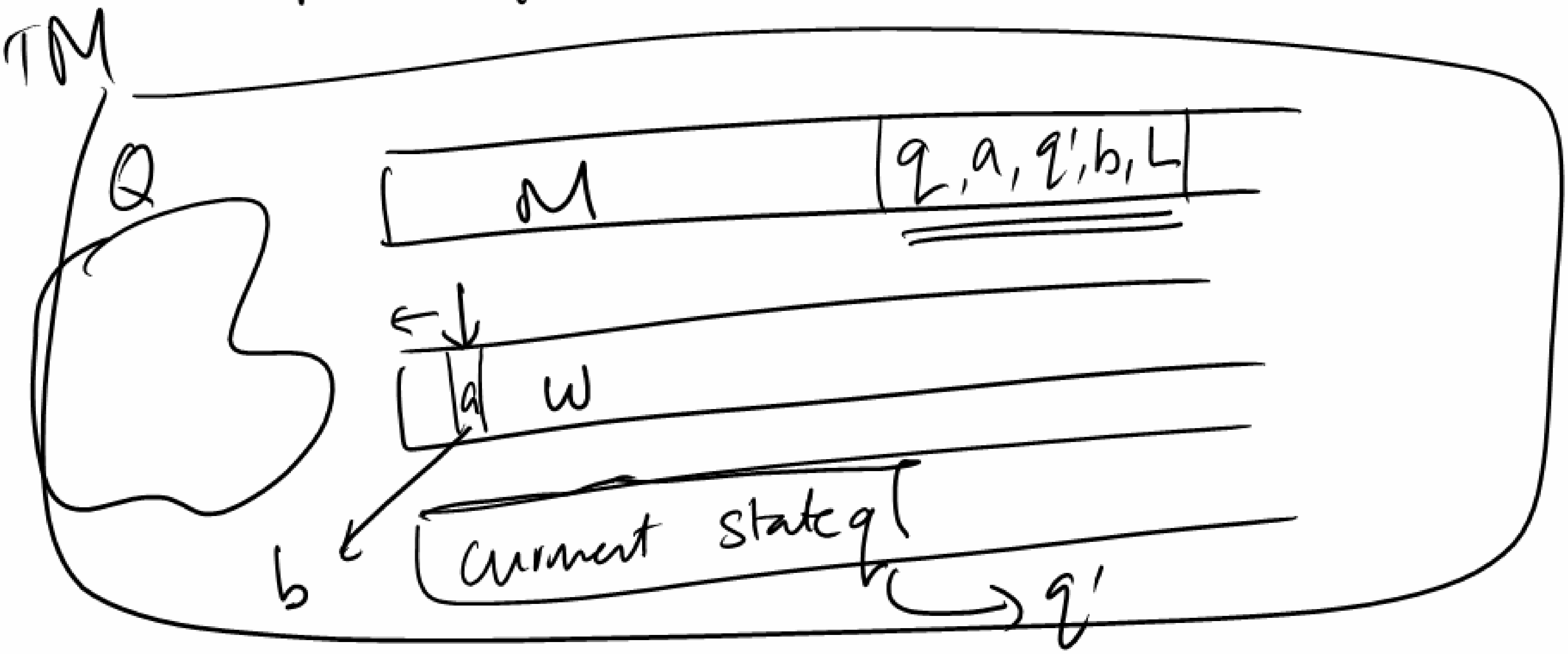
$WC_{TM} = \{ \langle M, k \rangle \mid M \text{ has } k \text{ transitions} \}$

is (trivially) decidable.

But <u>remember</u>

must check $M$ is a TM.

$$\text{Mem}_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \text{ i.e. } w \in L(M) \}$$




Accept if M accepts (i.e. current state is $q_{acc}$)

Reject if M rejects.

$\therefore$ $\text{Mem}_{TM}$ is recognizable

But not decidable!