Note Title 11/28/2006

Fun undecidable problems

Post's Corresponce Problem

$\left[\dfrac{ab}{bc}\right]$ is a domino.

$$\left\{ \begin{bmatrix} b \\ ca \end{bmatrix}, \begin{bmatrix} a \\ ab \end{bmatrix}, \begin{bmatrix} ca \\ a \end{bmatrix}, \begin{bmatrix} abc \\ c \end{bmatrix} \right\}$$

Match: $\begin{bmatrix} a \\ ab \end{bmatrix} \begin{bmatrix} b \\ ca \end{bmatrix} \begin{bmatrix} ca \\ a \end{bmatrix} \begin{bmatrix} a \\ ab \end{bmatrix} \begin{bmatrix} abc \\ c \end{bmatrix}$

$$\left\{ \begin{bmatrix} ab \\ b \end{bmatrix}, \begin{bmatrix} bc \\ ab \end{bmatrix}, \begin{bmatrix} c \\ abc \end{bmatrix} \right\}$$

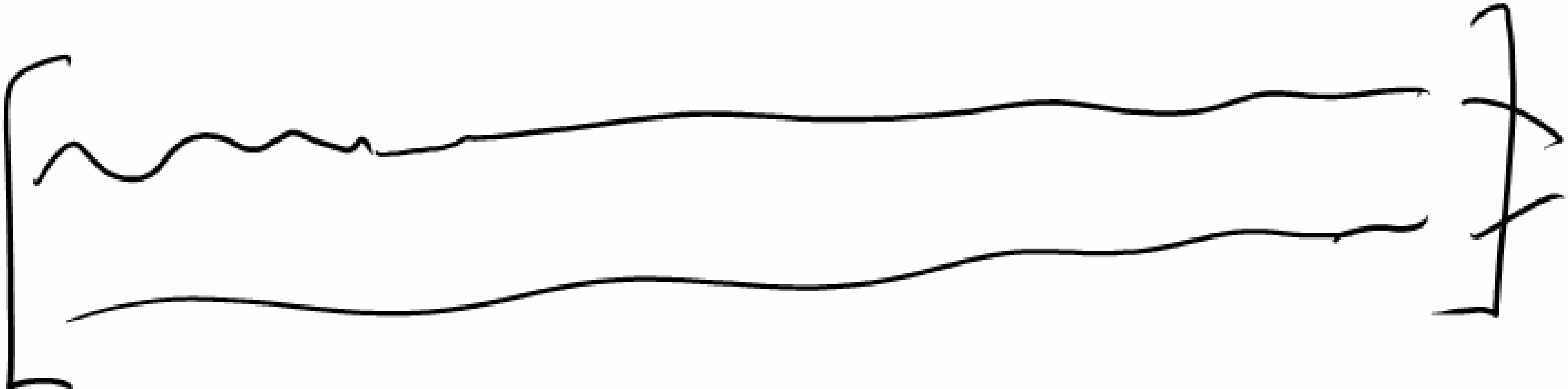
$$\left\{ \begin{bmatrix} [abc] \\ [a] \end{bmatrix} \begin{bmatrix} [bc] \\ [b] \end{bmatrix}, \begin{bmatrix} abab \\ a \end{bmatrix} \right\}$$

$$PCP = \{ \langle P \rangle \mid P \text{ has a match} \}$$

$$MPCP = \{ \langle P, d_0 \rangle \mid P \text{ has a match starting with domino } d_0 \in P \}$$

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$$

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$



I. $\left[\frac{\#}{\# \underline{\underline{q_0}}\, w_1 w_2 \ldots w_n}\right] \in P$ & is initial d_0.

II. If $\delta(q,a) = (r,b,R)$

$$\left[\frac{q\,a}{b\,R}\right]$$

III. If $\delta(q,a) = (r,b,L)$

$$\boxed{\frac{c\,q\,a}{r\,c\,b}}$$

$\underbrace{\quad}_{w_1} c\,q\,a \underbrace{\quad}_{w_2}$

$\underbrace{\quad}_{w_1} r\,c\,b \underbrace{\quad}_{w_2}$

IV. $\left[\frac{\#}{\#}\right] \qquad \left[\frac{\#}{\sqcup\,\#}\right]$

V. $\left[\frac{a}{a}\right] \qquad \forall a \in \Gamma$

VI. $\left[\frac{a\, q_{acc}}{q_{acc}}\right] \left[\frac{q_{acc}\, a}{q_{acc}}\right]$

$\dfrac{c_1 \quad c_2 \quad c_3 \ldots c_7}{c_1 \quad c_2 \quad c_3 \ldots c_7 \quad \underline{\underline{c_8}}}$

q_{acc}.

VII. $\left[\frac{q_{acc}\,\#\#}{\#}\right]$

$$\begin{bmatrix} \# & \\ \# q_0 0100\# \end{bmatrix} \begin{bmatrix} q_0 0 \\ 2 q_7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\delta(q_0, 0) = (q_7, 2, R)$$

$$\begin{bmatrix} q_0 0 \\ 2 q_7 \end{bmatrix}$$

$$\begin{bmatrix} \# & \\ \# q_0 0100\# \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \# \\ \# \end{bmatrix}$$

a_0

$$\begin{bmatrix} q_0 1 \\ 2 q_3 \end{bmatrix}$$

$$\begin{bmatrix} \# & \\ \# q_0 0100\# \end{bmatrix} \begin{bmatrix} q_0 0 \\ 2 q_7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \# \\ \# \end{bmatrix}$$

$$\delta(q_7, 1) = (q_5, 0, L)$$

$$\boxed{\begin{matrix} 2 q_7 1 \\ q_5 2 0 \end{matrix}} \quad \begin{bmatrix} 1 q_7 1 \\ q_5 1 0 \end{bmatrix} \quad \begin{bmatrix} 0 q_7 1 \\ q_5 0 0 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 2 q_7 1 \\ q_5 2 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \# \\ \# \end{bmatrix}$$

q_5 acc

$$\hookrightarrow \begin{bmatrix} q_5 2 \\ q_5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \# \\ \# \end{bmatrix}$$

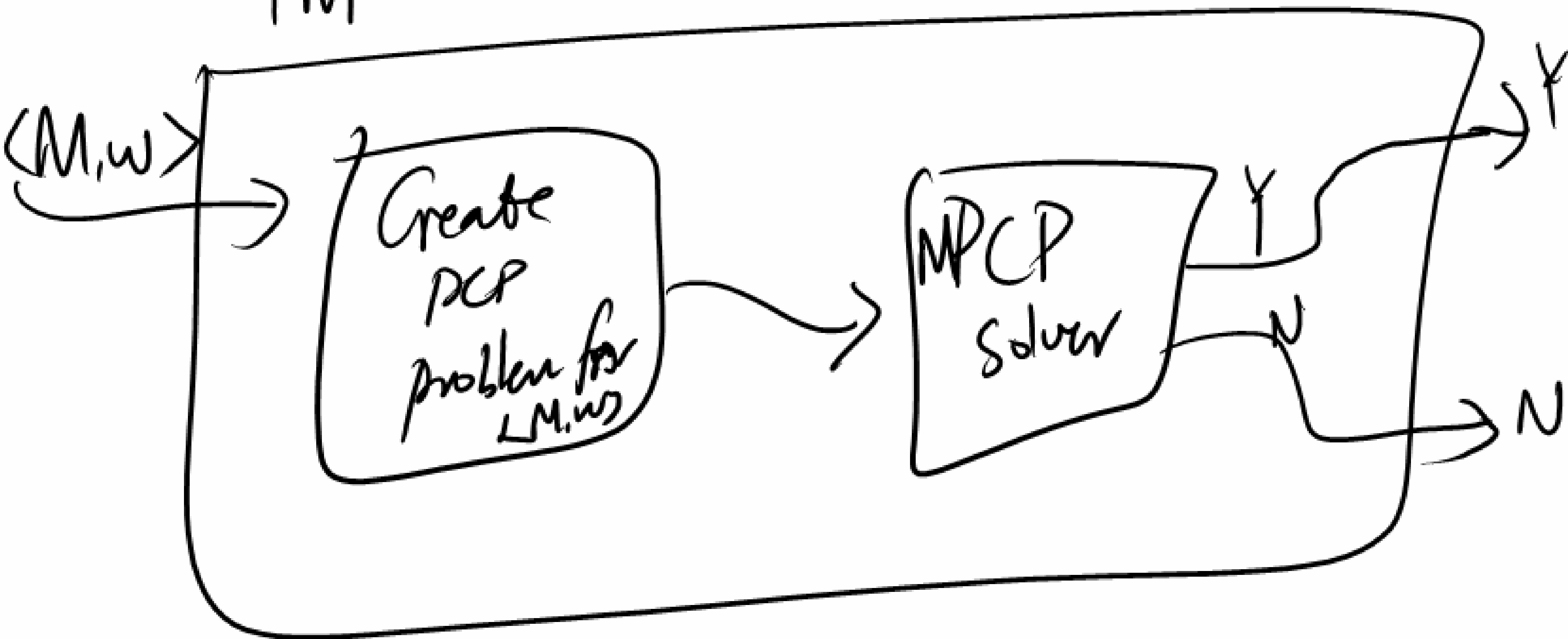
$$\begin{bmatrix} q_5 2 \\ q_5 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} q_5 0 \\ q_5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \# \\ \# \end{bmatrix}$$

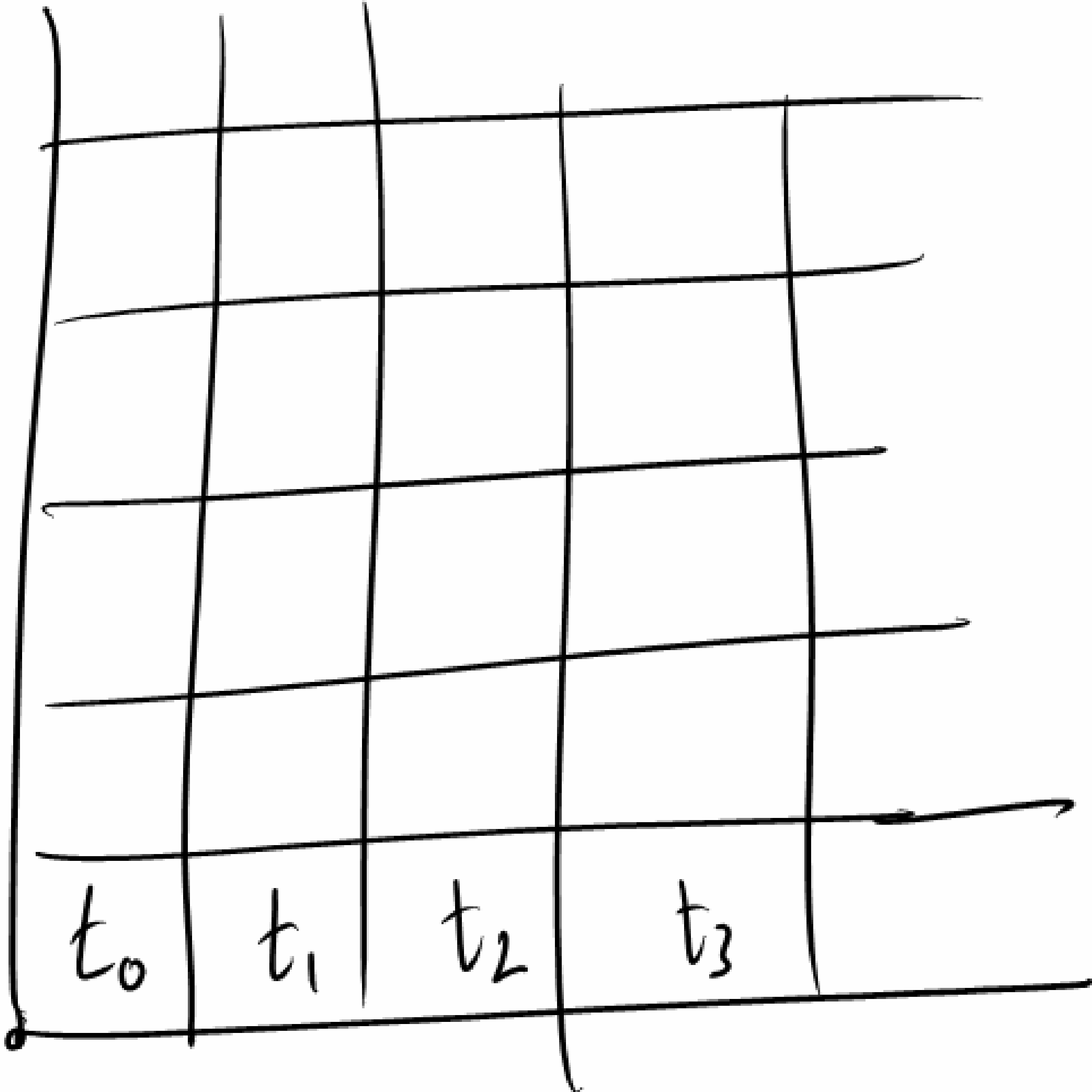
$\vdots$

$$\hookrightarrow \begin{bmatrix} \\ q_5 \end{bmatrix} \begin{bmatrix} \\ \# \end{bmatrix} \longrightarrow \begin{bmatrix} q_5 \#\# \\ \# \end{bmatrix}$$

A_{TM} : $\langle M, w \rangle$



Tiles T



PCP is undecidable

we know
MPCP is undecidable

MPCP problem $P = \{ d_0, d_1, \dots d_k \}$

d_0 is initial domino

$^*u = u_1 \dots u_k$

$* u = * u_1 * u_2 * \dots u_k$

$u * = u_1 * u_2 * \dots u_k *$

$* u * = * u_1 * u_2 \dots u_k *$

$P = \left\{ \begin{bmatrix} t_1 \\ b_1 \end{bmatrix} \begin{bmatrix} t_2 \\ b_2 \end{bmatrix} \dots \begin{bmatrix} t_k \\ b_k \end{bmatrix} \right\}$ — MPCP problem

$P' = \left\{ \begin{bmatrix} * t_1 \\ * b_1 * \end{bmatrix}, \begin{bmatrix} * t_1 \\ b_1 * \end{bmatrix}, \begin{bmatrix} * t_2 \\ b_2 * \end{bmatrix}, \dots \begin{bmatrix} * t_k \\ b_k * \end{bmatrix}, \begin{bmatrix} * \diamond \\ \diamond \end{bmatrix} \right\}$

(P, d_0) has an MPCP solution
iff P' has a PCP solution.

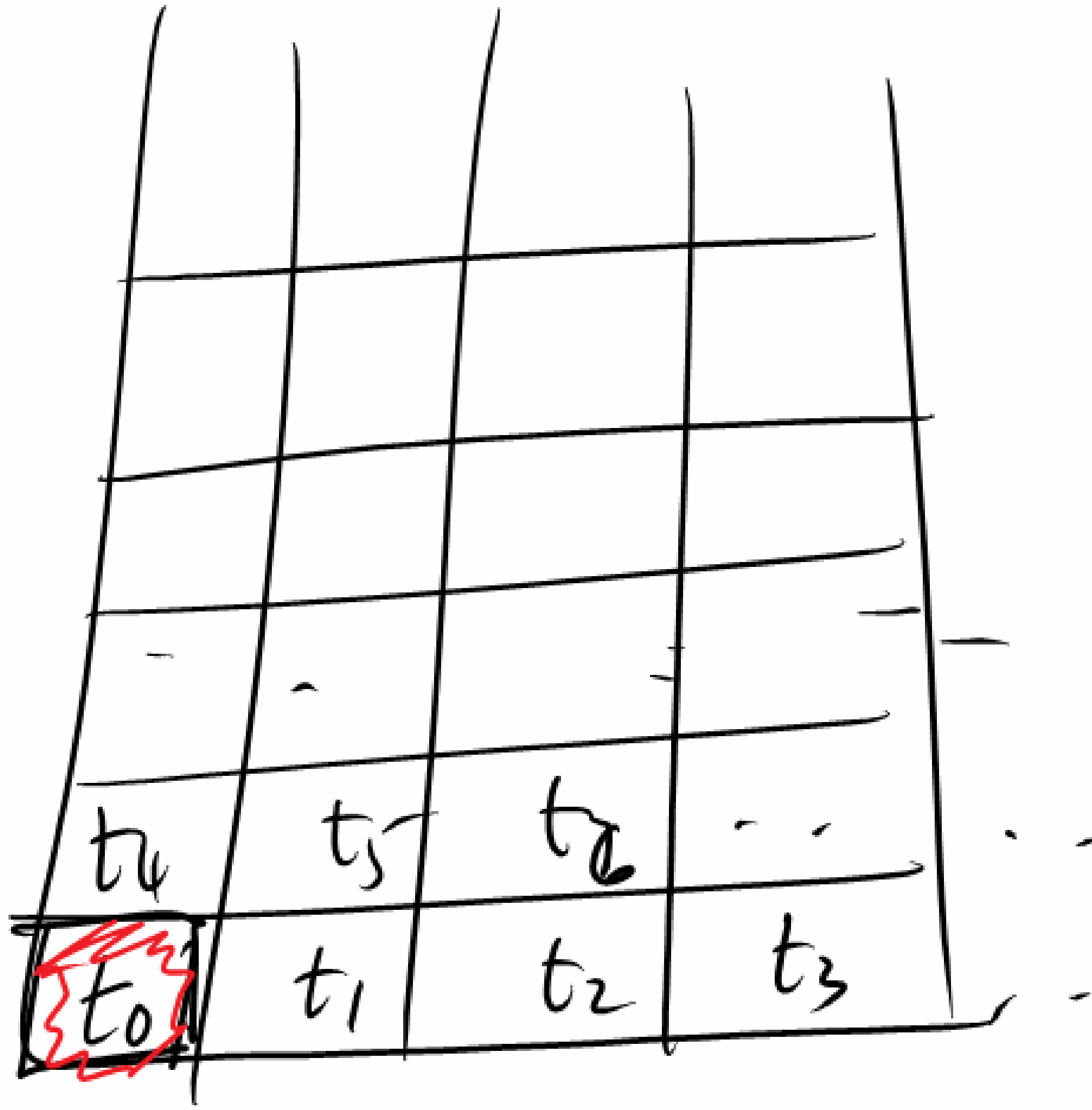
PCP is undecidable

T

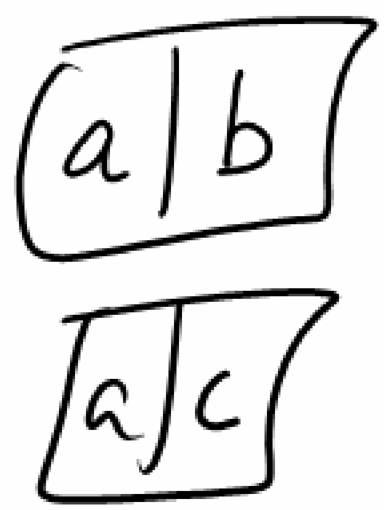
- t_0 should be in $(0,0)$

$H \subseteq T \times T$

$V \subseteq T \times T$



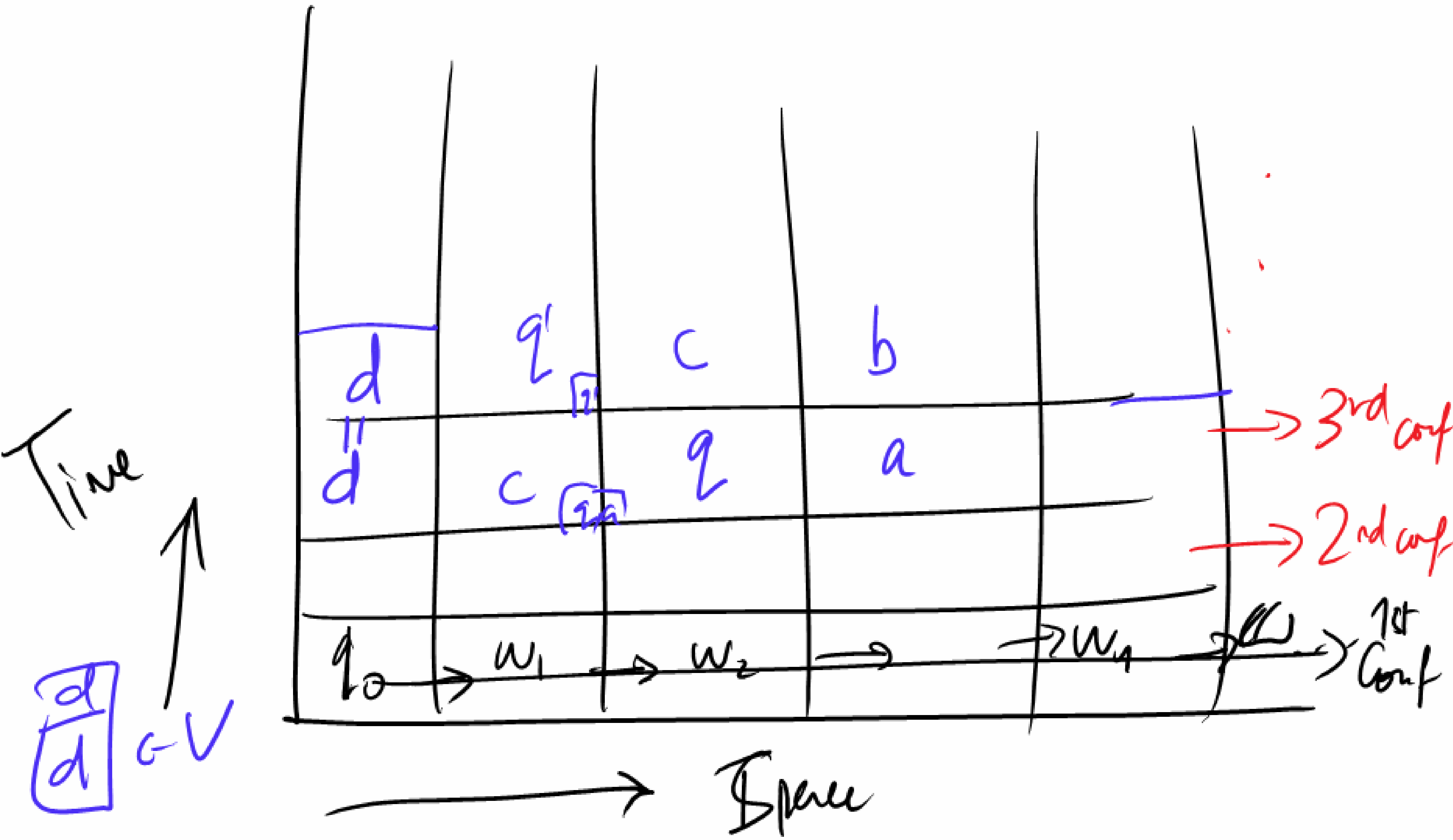
$\underline{(a,b)} \in H$



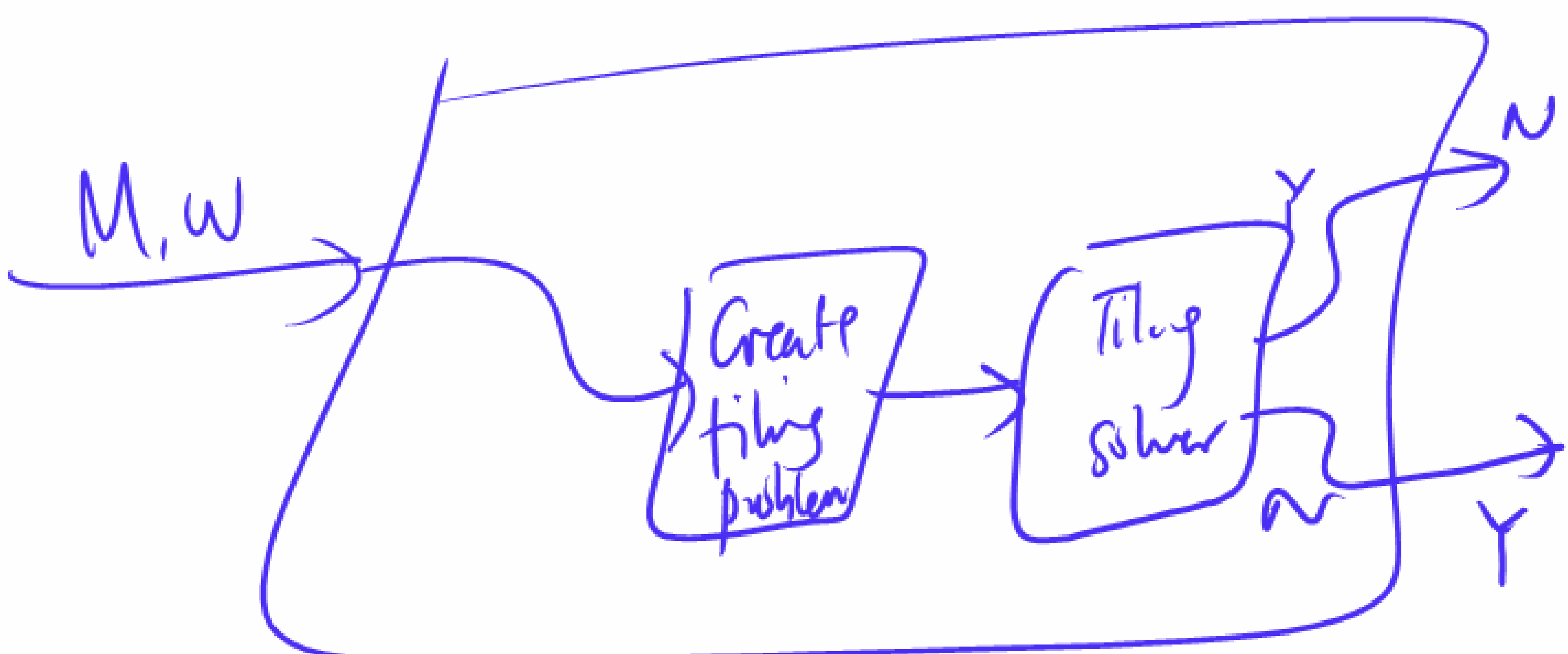
__Question__ Given (T, t_0, H, V)

is the entire quadrant of the plane tilable?

16 tiles can create aperiodic tiling.



TM M halts on w iff there is no tiling



PCP is Turing-recognizable

Tiling is <u>not</u> Turing recognizable.