CS 273
Lecture 2

Hw 0 is out

due next ~~Monday~~ Tuesday

Read 1.1 for Thursday

Proofs

- style
- outline
- details

Details

- special cases

  is $0 \in \mathbb{N}$? no

  $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$

- follow directions
- look up definitions
- justify steps in proof
- introduce new variables

Common Proof Types

- constructive
- contradiction
- induction

$$A = B$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

Numerical induction

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Numerical recursive definition

Fibonacci #'s

$$f_1 = f_2 = 1$$

$$f_k = f_{k-1} + f_{k-2}$$

Proof by induction on k

Base [trivial]

Induction

Assume true for $k = n$

Show true for $k = n+1$

More general induction

- multiple base cases
- "strong" induction
- on non-numerical objects

Strong induction

Theorem: Any integer ≥ 2 ~~is~~ can be written as the product of primes

Proof by induction on n

Base: $n=2$ 2 is prime ✓

Induction:

Assume all ~~number~~ integers $< k$ are products of primes

~~Let~~ Consider k.

2 cases

1) k is prime $\rightarrow$ we're done

2) k is composite

That means

$k = pq$ where

p, q are integers

≥ 2 (~~and $< k$~~)

Since $p \geq 2$, $q < k$

since $q \geq 2$, $p < k$

Since $p, q < k$, p is the product of primes and so is q.

Therefore $k = pq$ is a product of primes

$\square$ QED

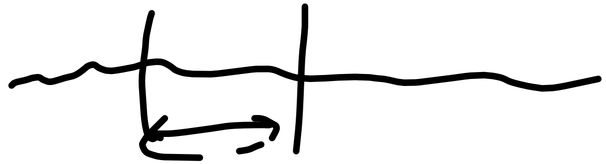
Strategy for Prouts

- Consider a big object size k
- Remove a bit or split it up
  $\Longrightarrow$ smaller objects

- Are smaller objects size $k-1$?

Non-numerical objects

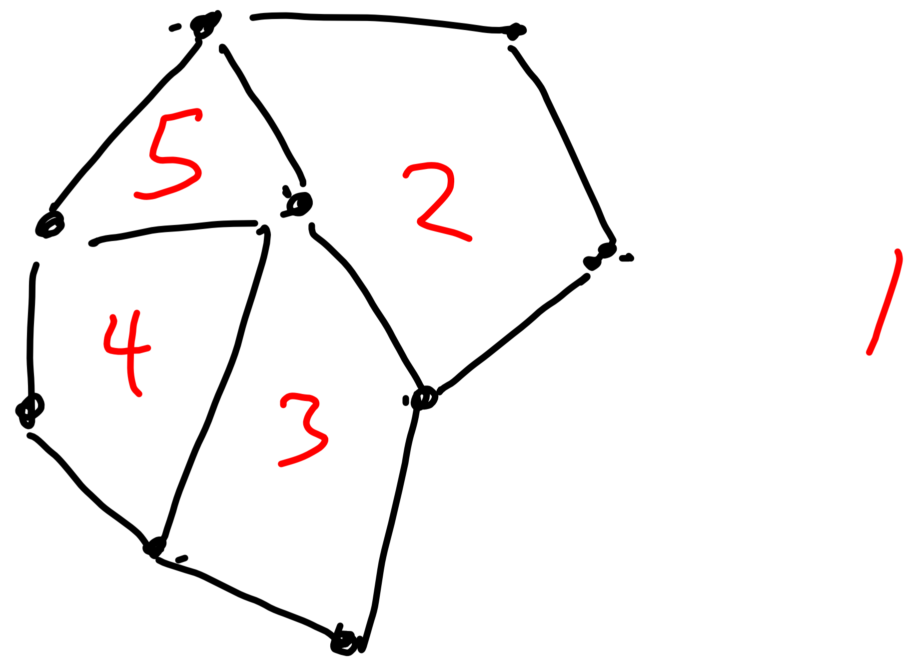
- Trees   # of nodes
- Strings ε $\in$ length



- graphs   # edges
  # vertices
- Sequences of rule applications
  # of rules applied

Example: Euler's Formula

$$V - E + F = 2$$



Let G be a connected planar graph

Then $V - E + F = 2$ where

$V = \#$ vertices in G

$E = \#$ edges in G

$F = \#$ faces in G

Proof: Induction on E (# of edges)

Base: $E = 0$

G looks like •

So $E = 0$ $F = 1$

$V = 1$

Induction:

Assume true for all graphs with $< k$ edges.

Let G be a graph with k edges.

Pick an edge e in G.

Case 1)

collapse e to a single vertex

E gets one smaller
F stays same
V gets one smaller

Case 2)

e joins a vertex to itself

delete e

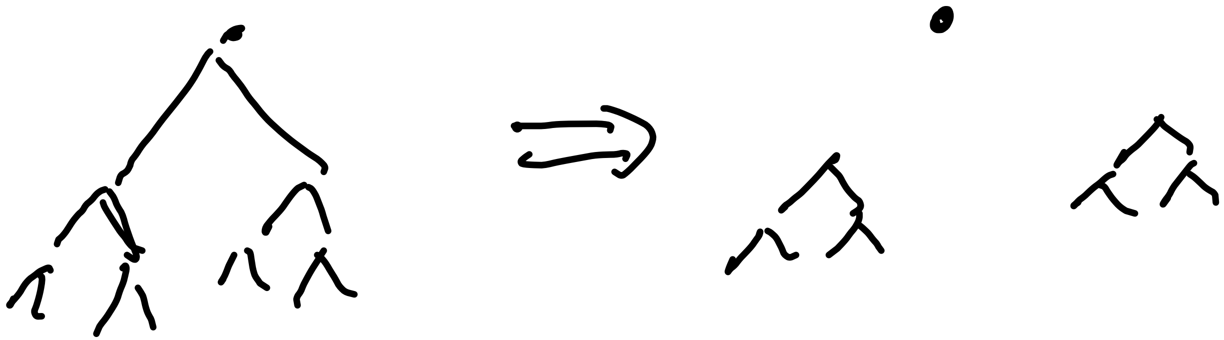
F gets one smaller

E gets one smaller

V stays same

Graph: remove edge or vertex

Trees: split at root



Strings: remove 1st or last char
or prefix or suffix

Rule application: remove last rule

Define S as follow
$\subseteq \mathbb{Z}$

1) $3 \in S$

2) If x and $y \in S$
Then $x - y \in S$

3) Nothing else is in S
- S is the smallest S satisfying 1 & 2
- Everything in S is made by applying rules 1 & 2 a finite # of times

Claim:

$$S = \{\text{all multiples of } 3\} = 3\mathbb{Z}$$

Need To show

$$S \subseteq 3\mathbb{Z}$$

and

$$3\mathbb{Z} \subseteq S$$

To show $3\mathbb{Z} \subseteq S$

Proof by Induction on k = absolute value of the number in $3\mathbb{Z}$

to show that $3k \in S$

and $-3k \in S$

$\forall k \geq 0$

To show $S \subseteq 3\mathbb{Z}$

$\underbrace{(1) \quad (2) \quad (2) \quad \ldots}_{k} \quad \underbrace{(2)}_{1}$

3   3-3=0   3-0   -

Induction on # rules applied