Context-free Languages

and

Pushdown Automata

HW3. Part I pick-up at end of lecture.

Context-free grammars

Non-terminals , Terminals

$$G \;\Big|\; \begin{array}{l} S \longrightarrow \underline{0}\, S\, \underline{\underline{1}} \;\; | \;\; \varepsilon \\ S \longrightarrow \varepsilon \end{array}$$

S - initial non-terminal.

$$L = \{ \epsilon, 01, \underline{\underline{0011}}, \ldots \}$$
$$= \{ 0^n 1^n \mid n \geq 0 \}$$

$$S \longrightarrow 0\underline{\underline{S}}1 \longrightarrow \underline{\underline{0011}} \in \underline{\underline{L(G)}}$$

- Context free grammars

  — Context free languages

- $RL(\Sigma) \subseteq CFL(\Sigma)$

- Declarative style (CFG) | Like Reg Exp.

- Chomsky Normal Form

$$\boxed{\begin{array}{l} A \to BC \\ A \to a \end{array}}$$

Computable function.	TM	
CFL	PDA	CFG
Reg Lang	DFA = NFA	Reg Exp.
	Machine	Decl.

$RL \subseteq CFL$

RegExp. $\longrightarrow$ CFG

$a \longrightarrow$ $S \to a$

$\varepsilon \longrightarrow$ $S \to \varepsilon$

$\emptyset \longrightarrow$ S

$S \to S$

$T \to \dots$

$R_1 \cup R_2$

$S \to S_1$

$S \to S_2$

R_1: $S_1 \to \dots$

R_2: $S_2 \to \dots$

$R_1 \circ R_2$

$S \to S_1 S_2$

$\{ xy \mid x \in R_1,\ y \in R_2 \}$

R_1: $S_1 \to \ldots$

R_2: $S_2 \to \ldots$

$S \to S_1 S_2 \to x S_2 \to xy$

R_1^*

$S \to \varepsilon \mid S_1 \cdot S$

R_1: $S_1 \to \ldots$

$$S \to \varepsilon \mid (S) \mid S.S$$

$$(())()$$

$$S \to S.S \to (S).S$$

$$\to ((S))S \to (())S$$

$$\to (())(S)$$

$$\to (())()$$

Pushdown Automata

Motivation for CFGs:

Parsing

$$\Sigma = \{ (\, ,\,) \}$$

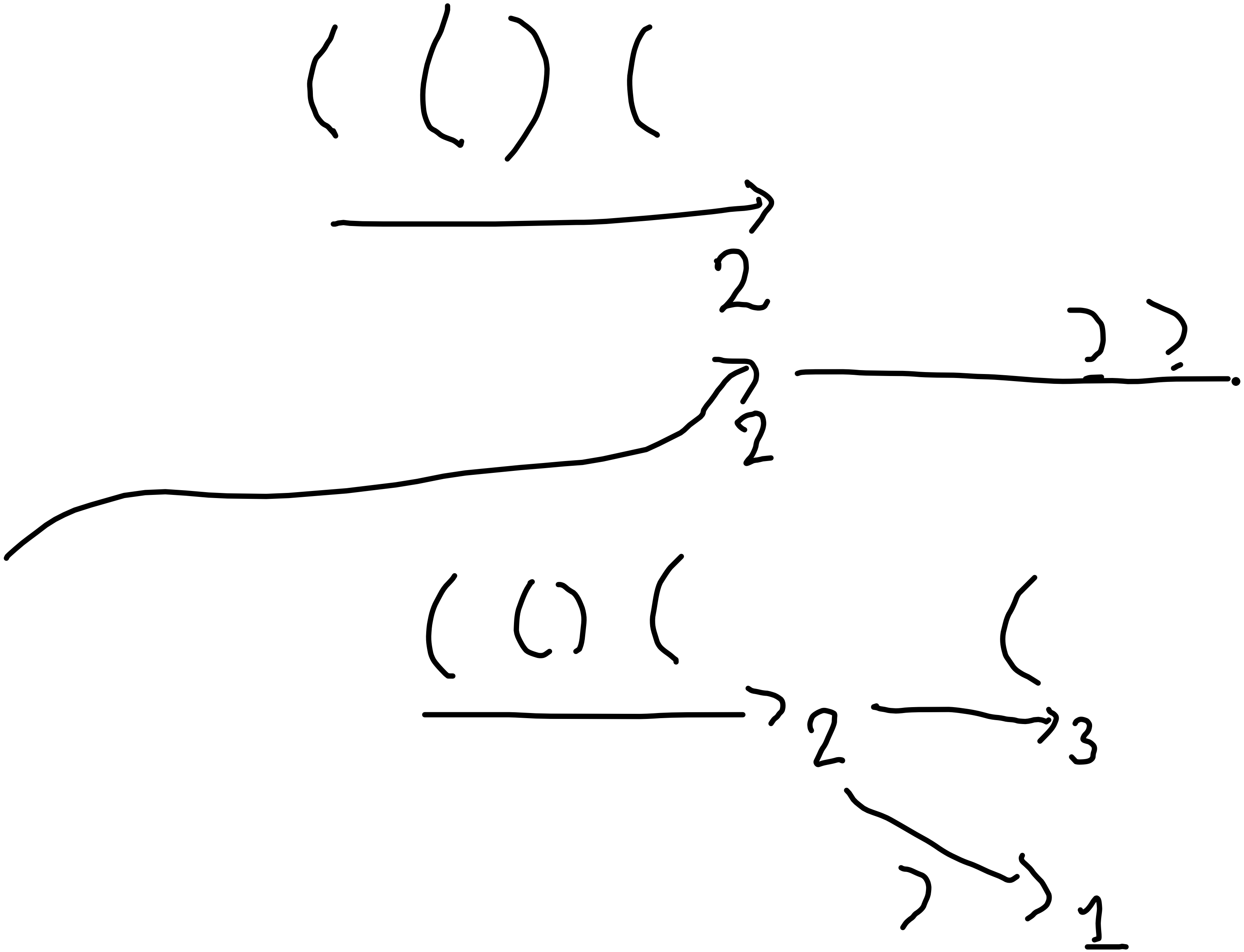
L be the well-bracketed words over Σ.

$(())() \in L$ $\qquad$ $((\notin L$

$\qquad\qquad\qquad\quad$ $()) \notin L$

$$L \cap \cancel{[()]^*}\; (^*)^*$$

$$= \{ (^n)^n \mid n \geq 0 \}$$

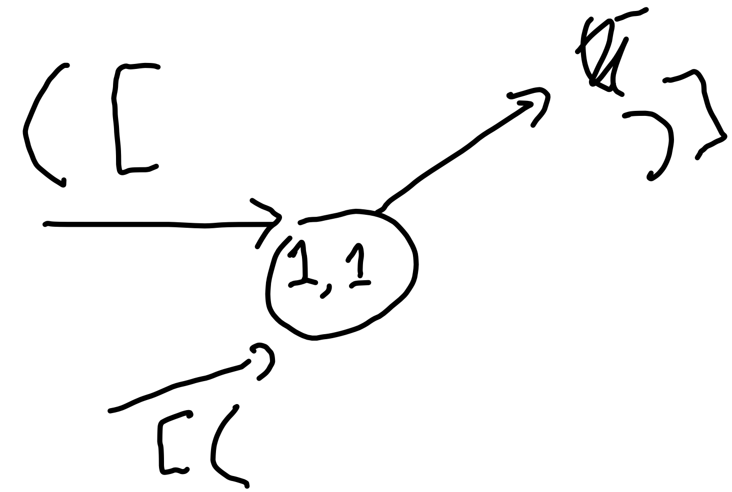

(( ) (
2
))
2
(( ) (
(
2
3
)
1

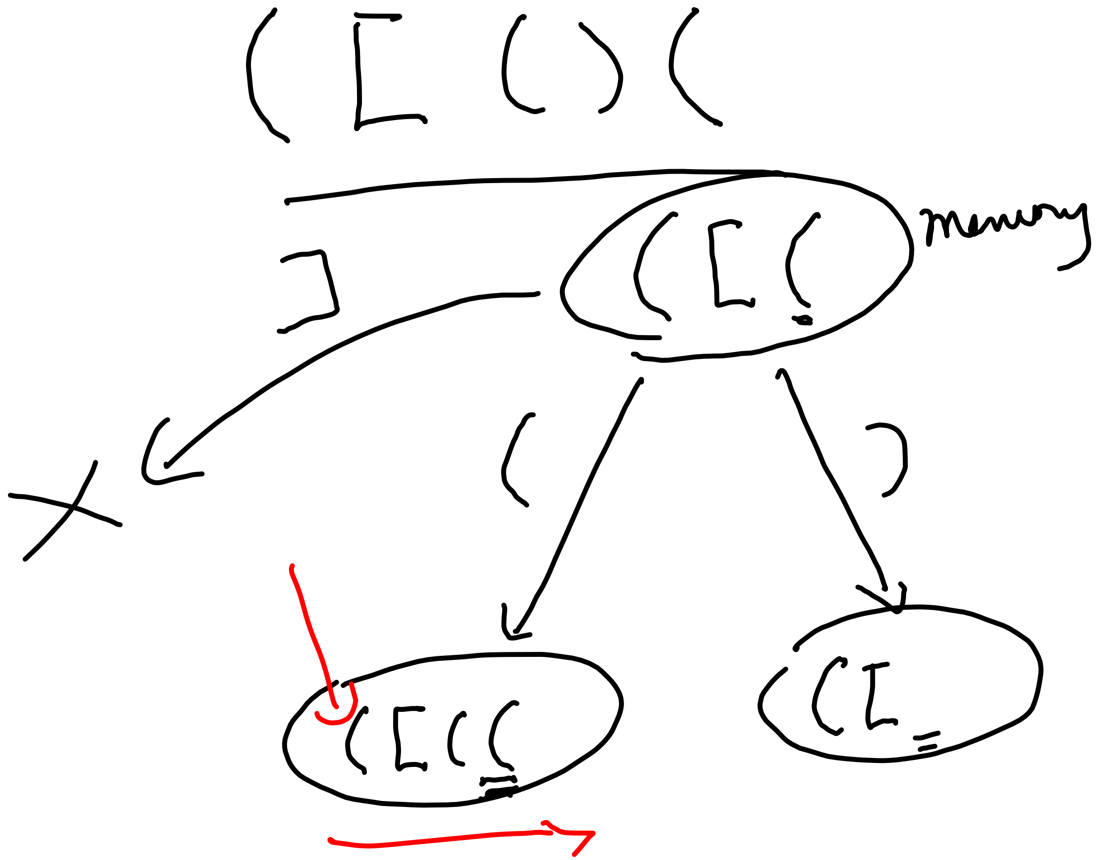
$$\Sigma = \{ (,), [,] \}$$

$() \in L$

$(] \notin L$

$([)]$ $\cancel{(]}$

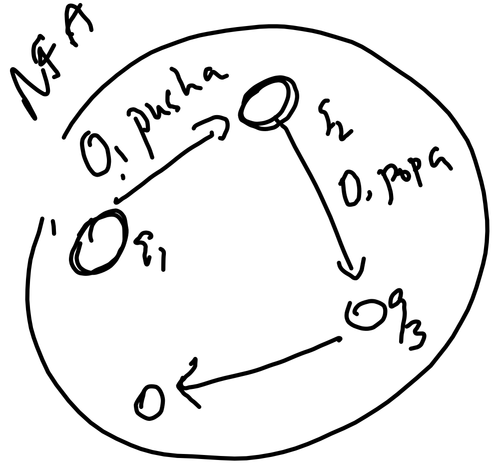


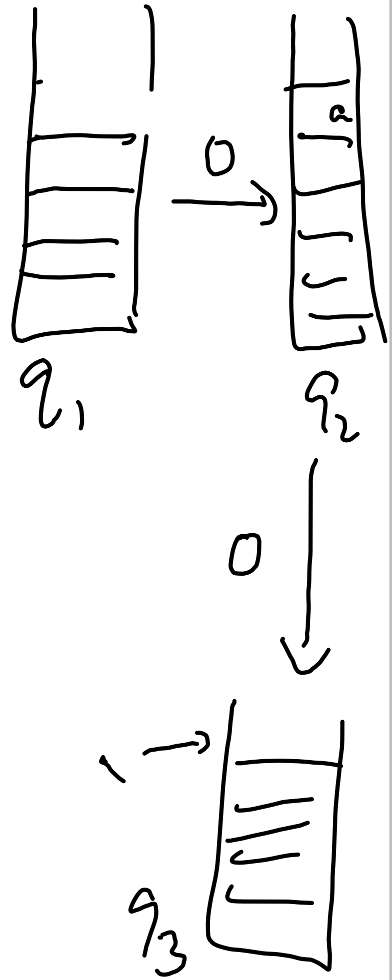


Memory needed: word

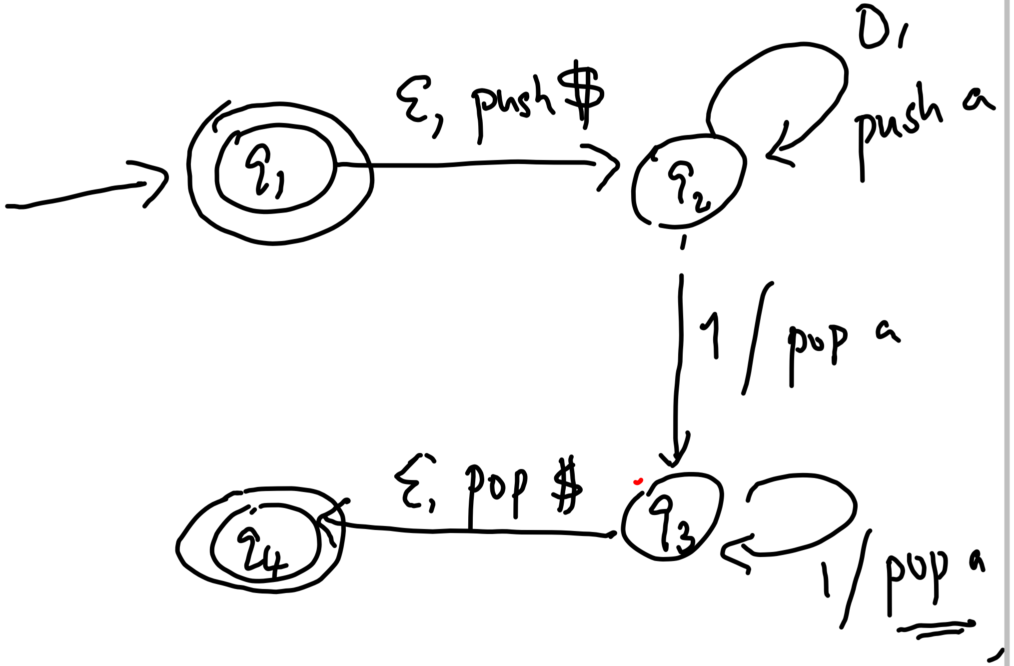
LIFO memory
STACK

Pushdown Automata = NFA + stack

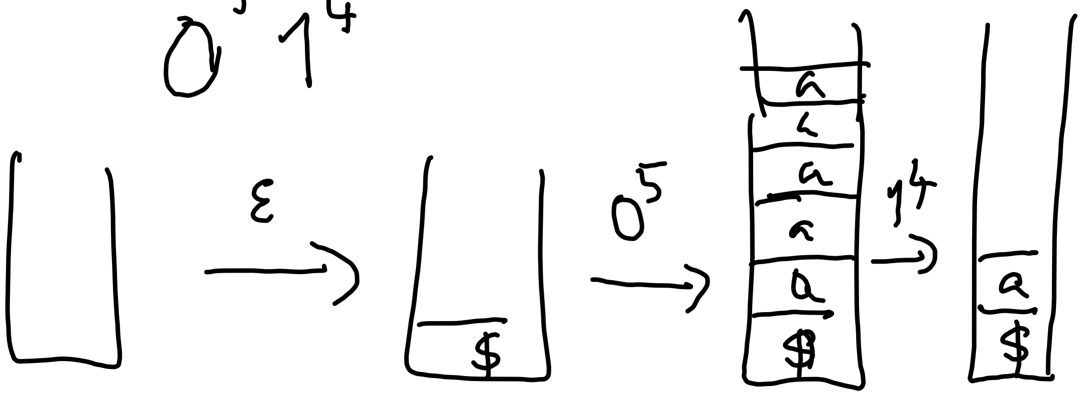




$$L = \{ 0^n 1^n \mid n \geq 0 \}$$



$0^5 1^4$

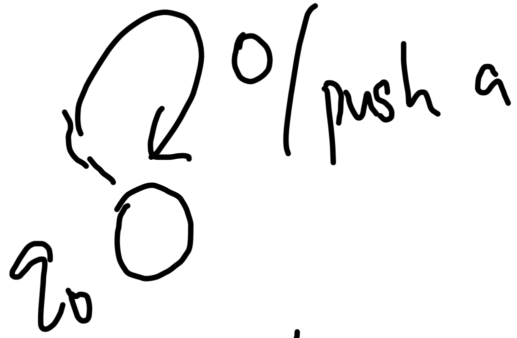


Γ - stack alphabet

Σ - input alphabet

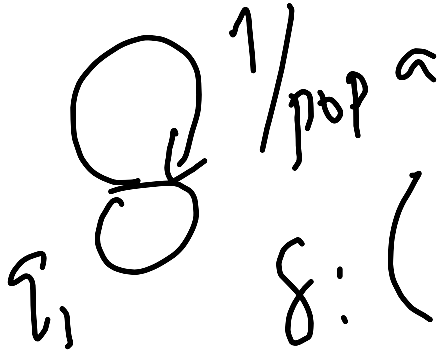
$$\Sigma_\varepsilon = \Sigma \cup \{\epsilon\}$$

$$\Gamma_\varepsilon = \Gamma \cup \{\epsilon\}$$



Σ_ε Γ_ε what should be on top of stack

$$\delta : (\underbrace{q_0}_{Q}, 0, \underline{\underline{\varepsilon}}) \longrightarrow \{(\underbrace{q_0}_{Q}, \underbrace{a}_{\Gamma_\varepsilon})\}$$



$$\delta : (q_1, 1, \underline{a}) \longrightarrow \{(q_1, \epsilon)\}$$

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\varepsilon \longrightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$$

Pushdown automaton is

a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$

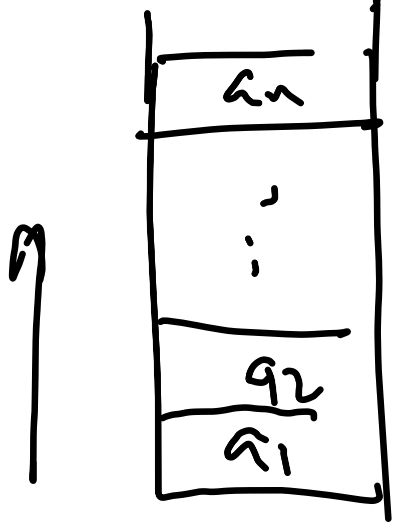
- Q is a finite set of states
- Σ - input alphabet (finite)
- Γ - stack alphabet (finite)
- $q_0 \in Q$ - initial state
- $F \subseteq Q$ - accept states

- δ - transition function.

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \to \mathcal{P}(Q \times \Gamma_\epsilon)$$

$$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

Stack is represented a word



$a_n a_{n-1} \cdots a_1$

(q, s) $\quad q \in Q \quad s \in \Gamma^*$

M accepts a word w if

w can be written as

$$w = w_1 w_2 \dots w_m$$

where each $w_i \in \Sigma_\epsilon$

and there is a sequence of states $r_0, r_1, r_2, \dots r_m \in Q$

and a sequence of strings

$$s_0, s_1, s_2, \dots s_m \in \Gamma^*$$

such that

1) $r_0 = q_0$, $s_0 = \epsilon$

2) $\forall i : 0 \leq i < m$

$$(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$$

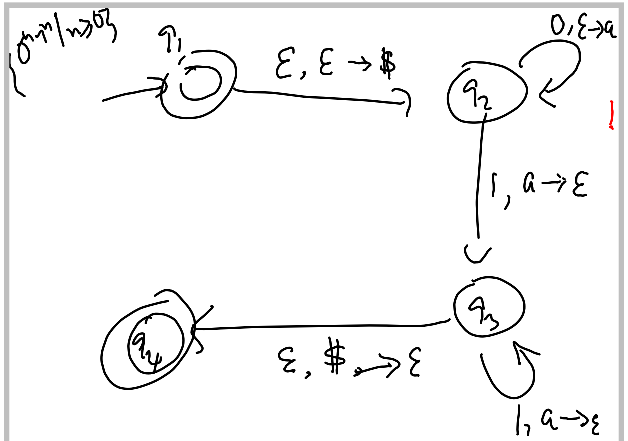
$$s_i = at \quad (\text{for some } t \in \Gamma^*)$$

$$s_{i+1} = bt$$

$a, b \in \Gamma_\epsilon$

3) $r_m \in F$

$a^i b^j c^k$ $\quad$ $i = j$ or $j = k$



$Q = \{q_1, q_2, q_3, q_4\}$

q_0 Initial state : q_1

$\Sigma = \{0, 1\}$

$\Gamma = \{\$, a\}$

$F = \{q_1, q_4\}$

$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow P(Q \times \Gamma_\varepsilon)$

Input	0			1			ε		
Stack	a	\$	ε	a	\$	ε	a	\$	ε
q_1	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		$\emptyset$			$\{(q_2, \$)\}$
q_2	$\emptyset$	$\emptyset$	$\{(q_2, a)\}$	$\{(q_3, \varepsilon)\}$					
q_3	$\emptyset$	$\emptyset$	$\emptyset$	$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

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