a homomorphism h
is a function That
maps characters to
strings

$$h : \Sigma \to \Gamma^*$$

Ex: $\Sigma = \{a, b, c, \#\}$

$\Gamma = \{m, p, k\}$

h: $a \to m$

$b \to mpp$

$c \to \varepsilon$

$\# \to k$

not There really

$$h(c\#baa) = \hat{\varepsilon}\, k\, \overbrace{mpp}^{b}\, mm$$

$$= kmppmm$$

$$L = \{ s^n t^n : n \geq 0 \}$$

Claim
L can't be regular.

Suppose L is regular.

Let $h: s \to 0$
$\qquad\quad t \to 1$

Then $h(L) = \{ 0^n 1^n : n \geq 0 \}$

Since L is regular and
regular lgs are closed
under homomorphism

Then $h(L)$ must be regular

But we've shown that
$h(L)$ isn't regular

\#

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$$L'' = \{ s^n \# t^n : n \geq 0 \}$$

$$h: s \to 0$$
$$t \to 1$$
$$\# \to \varepsilon$$ (# disappears)

$$h(L'') = \{ 0^n 1^n : n \geq 0 \}$$

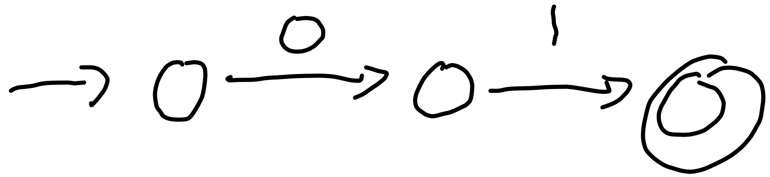
input n NFA

need to build DFA

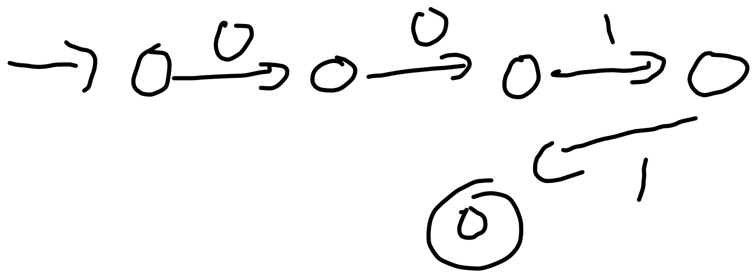
M_n recognizes

$\{0^n 1^n\}$ n fixed constant

$n = 1$



$n = 2$



in general

$$M_n = (Q_n, \Sigma, \delta_1, q_1, F_n)$$

$$Q_n = \{q_1, \ldots, q_{2n+1}\}$$

$$F_n = \{q_{2n+1}\}$$

$$\delta(q_i, 0) = \{q_{i+1}\}$$

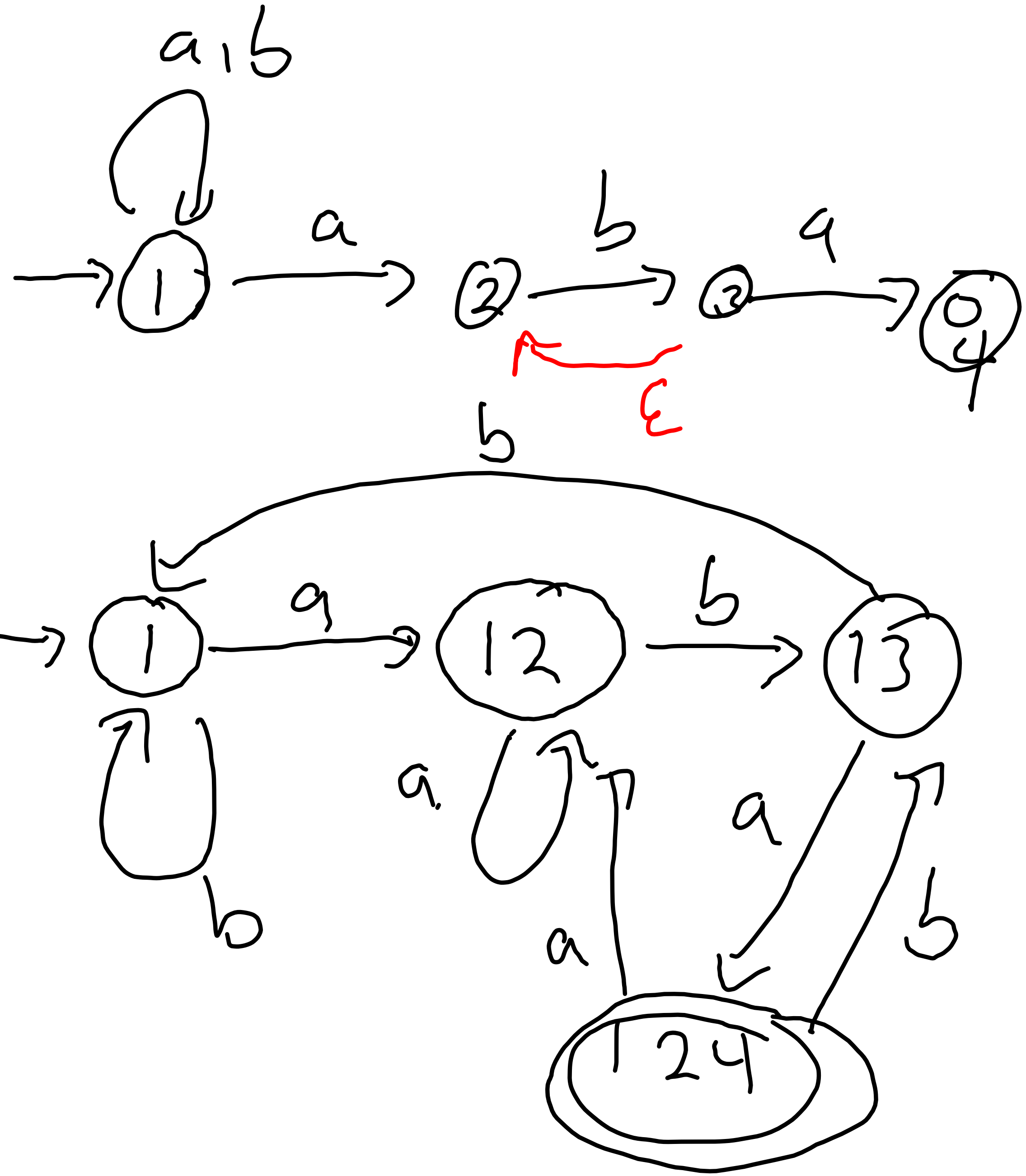
if $i \leq n$

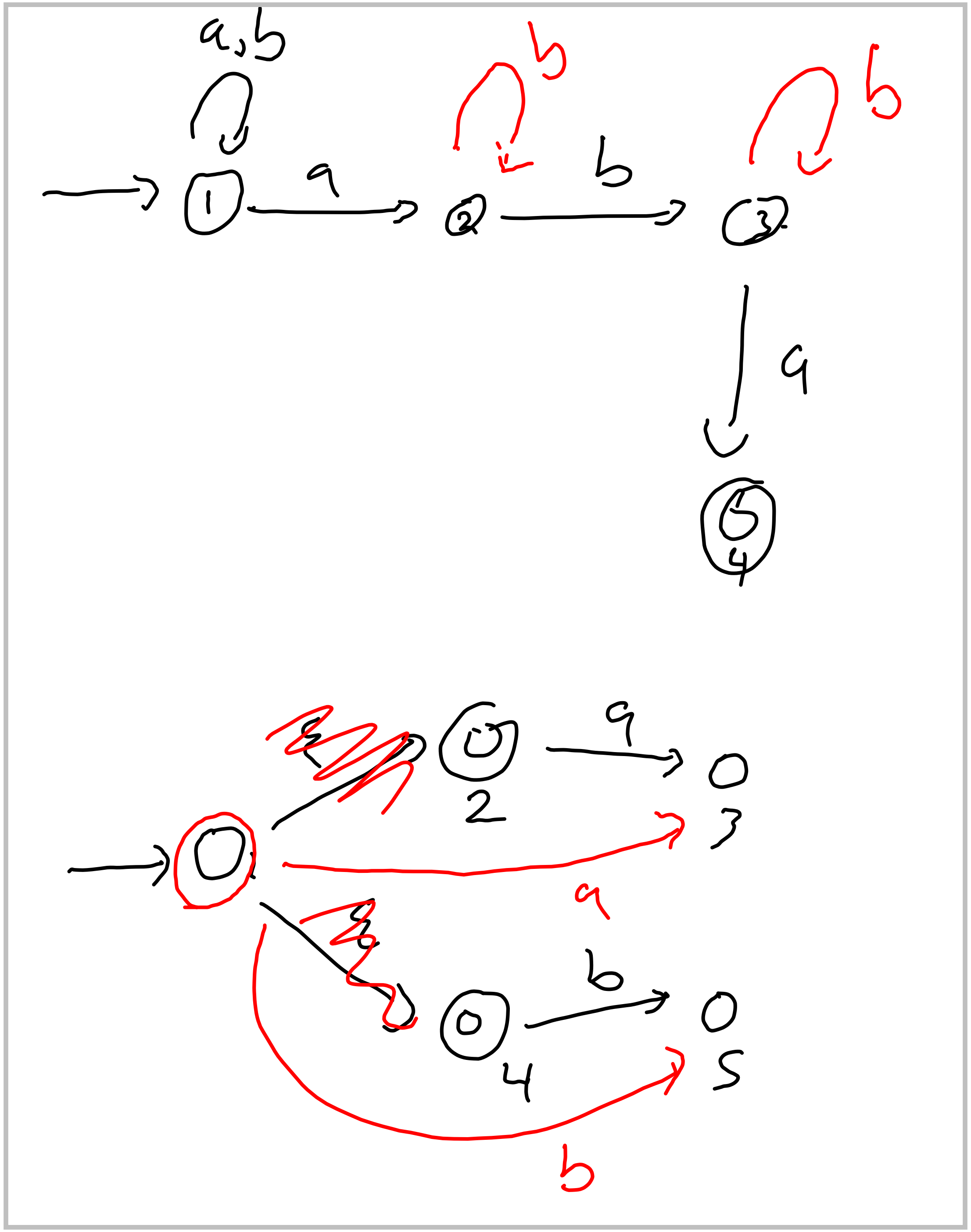
$\emptyset$ otherwise

$$\delta(q_i, 1) = \{q_{i+1}\}$$

if $n < i \leq 2n$

$\emptyset$ otherwise


a,b
a
b
a
1
2
3
4
ε
b
a
b
1
12
13
a
b
a
a
b
124



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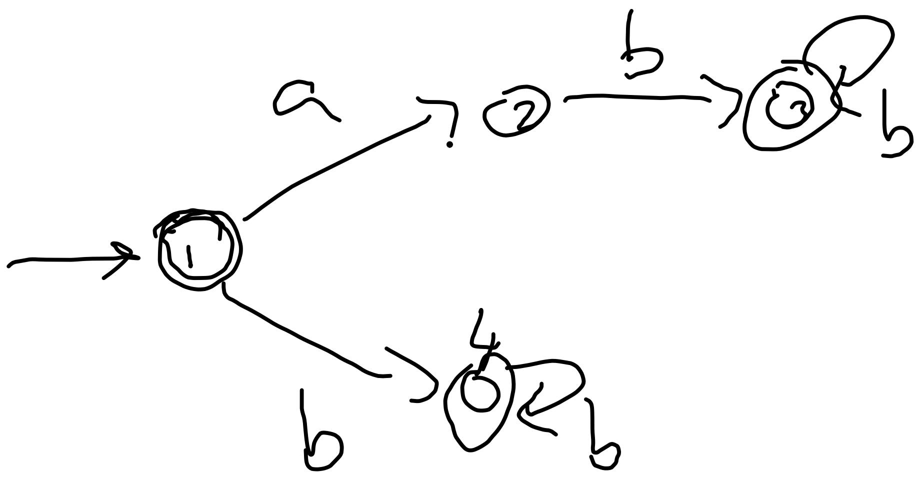


1. $abb^* \cup bb^*$

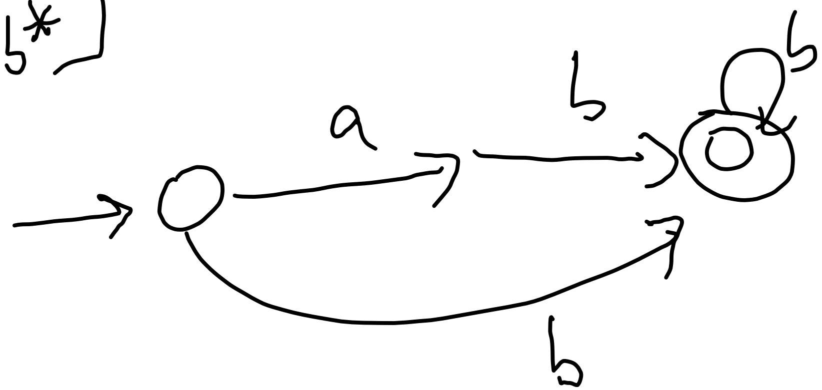
2. bb^*

3. b^*

4. b^*

abb^*

$\cup\ bb^*$



Generic construction

NFA $M = (Q, \Sigma, q_0, \delta, F)$

$\downarrow$

$M' = (Q', \Sigma, q_0', \delta', \cancel{F}\ \{f_*\})$

NFA $\longrightarrow$ NFA with only one final state

$Q' = Q \cup \{f_*\}$ (Assume $f_* \notin Q$)

$\delta'(q, a) = \delta(q, a)$ if $q \in Q$

$\delta'(q, \varepsilon) = f_*$ if $q \in F$

$\delta'(f_*, a) = \emptyset$