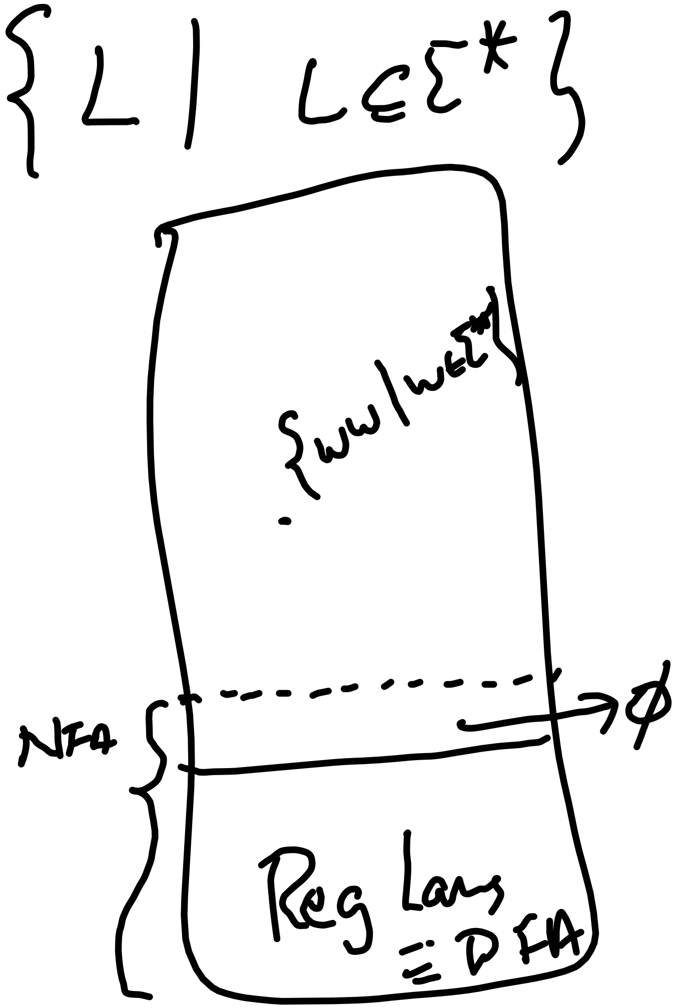
Nondeterministic and Deterministic Finite Automata

Regular Expressions

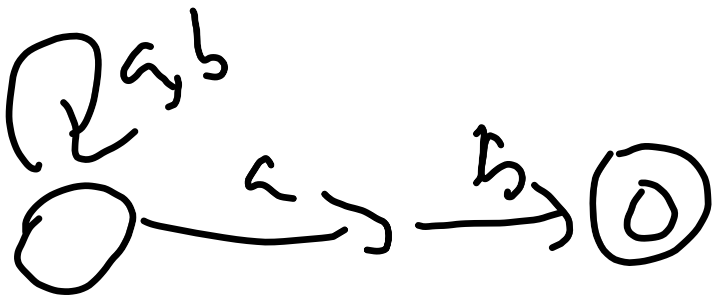
Every DFA is an NFA



Nondeterminism

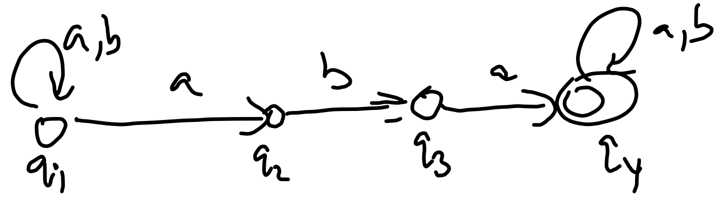
1) Mathematical Def — Don't think about it

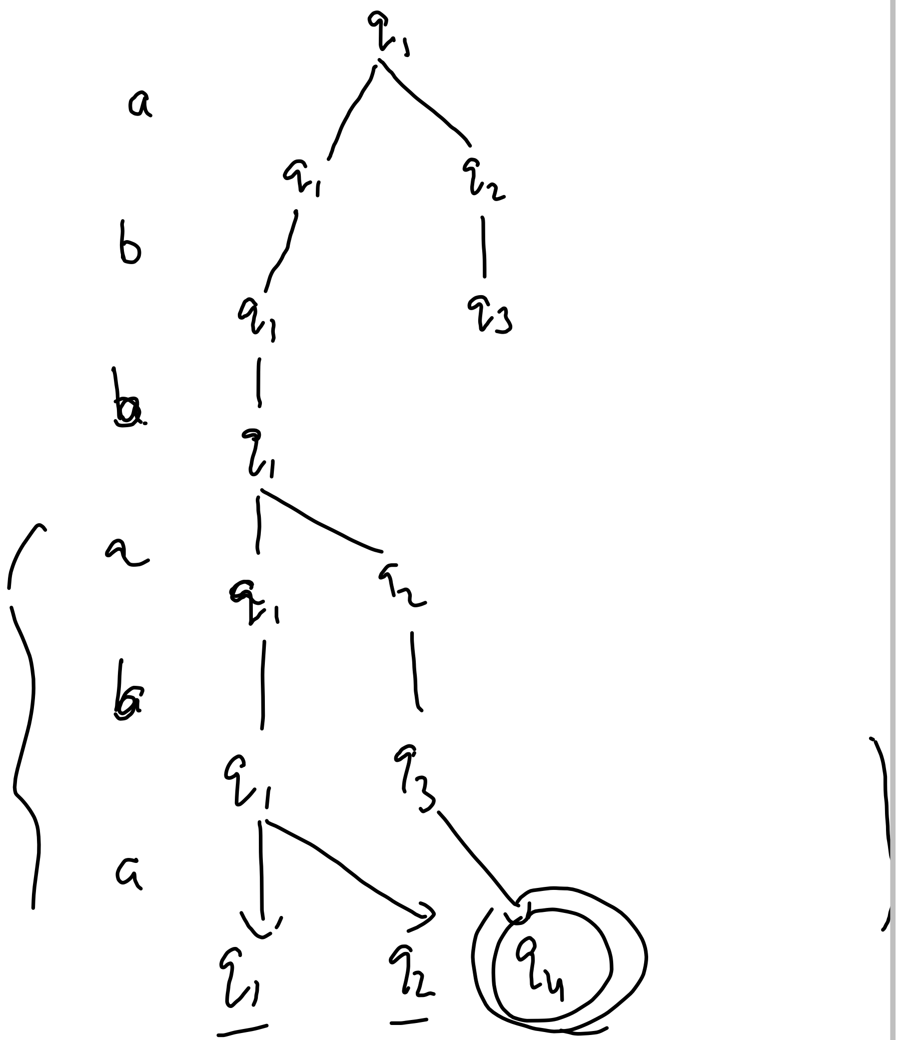
2) NFA "guesses but checks"

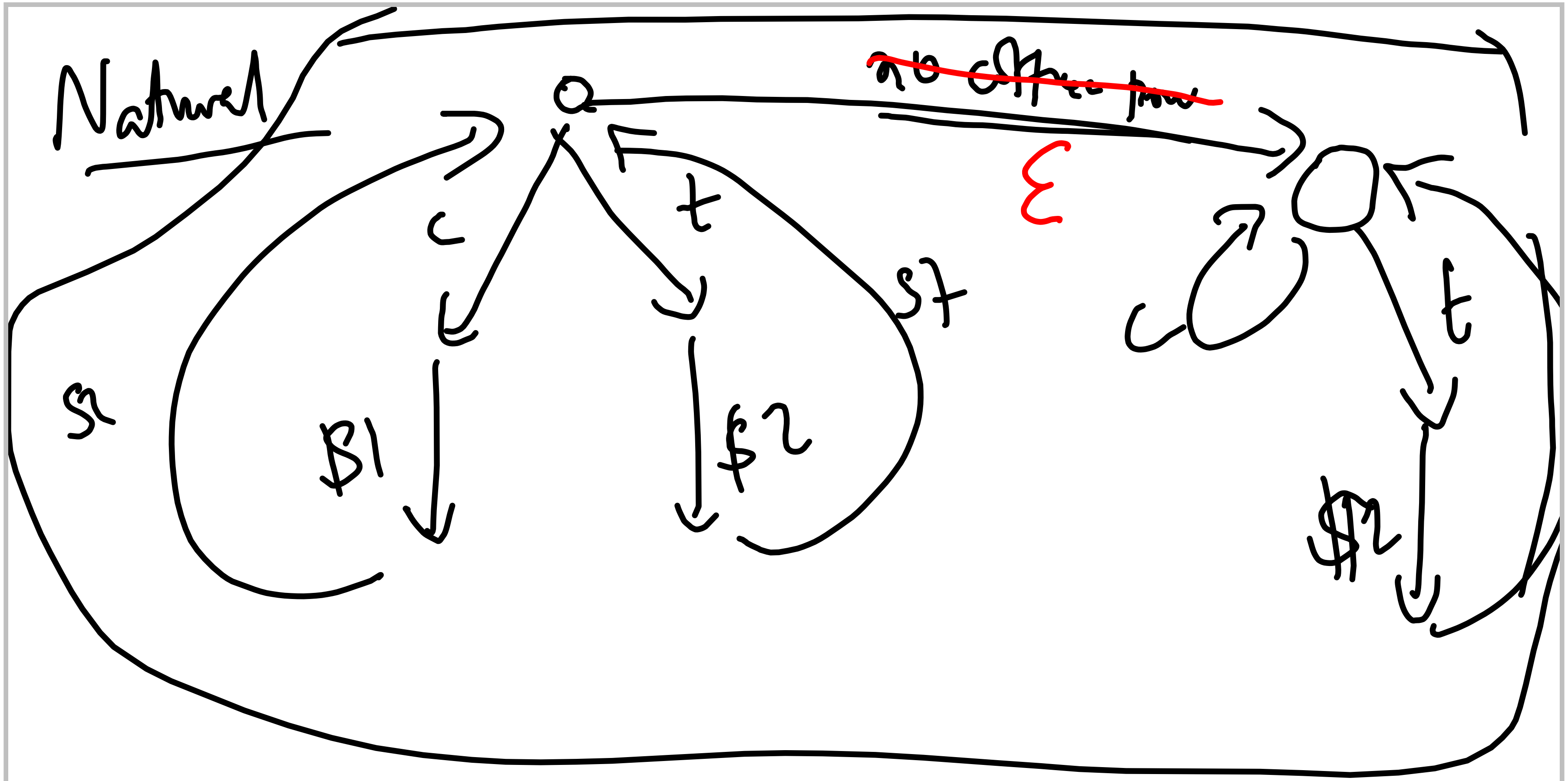


3)

3) NFA explores <u>all</u> computations.







c $1 sc

c c

NFAs are more compact than DFAs.

L

$w \in L$

A NFA L

B DFA L

$$|B| \gg |A|$$

Big picture

Machine model $\longrightarrow$ Nondet

M $\longrightarrow$ M

Finite automata $\longrightarrow$ NFA

DFA

Polytime Turing machine $\longrightarrow$ Nondet Turing machine running in polynomial time

$$P \stackrel{?}{=} NP$$

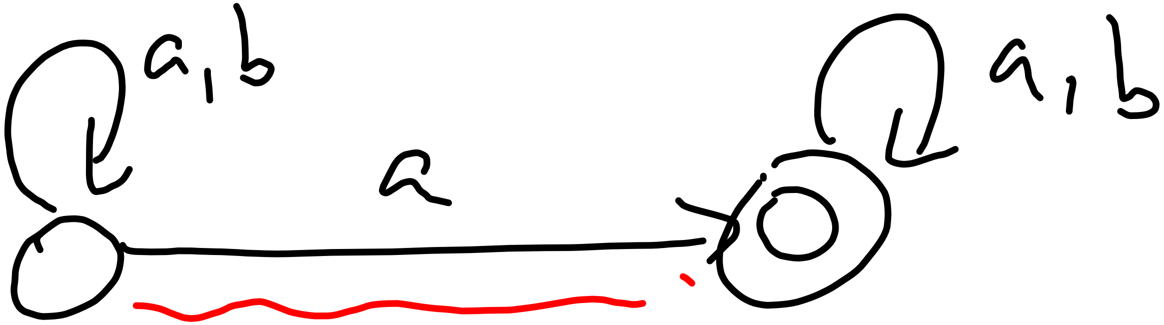
Thm. NFAs are equivalent to DFAs

For every NFA A

there is a DFA B s.t.

$$\mathcal{L}(A) = \mathcal{L}(B)$$

M

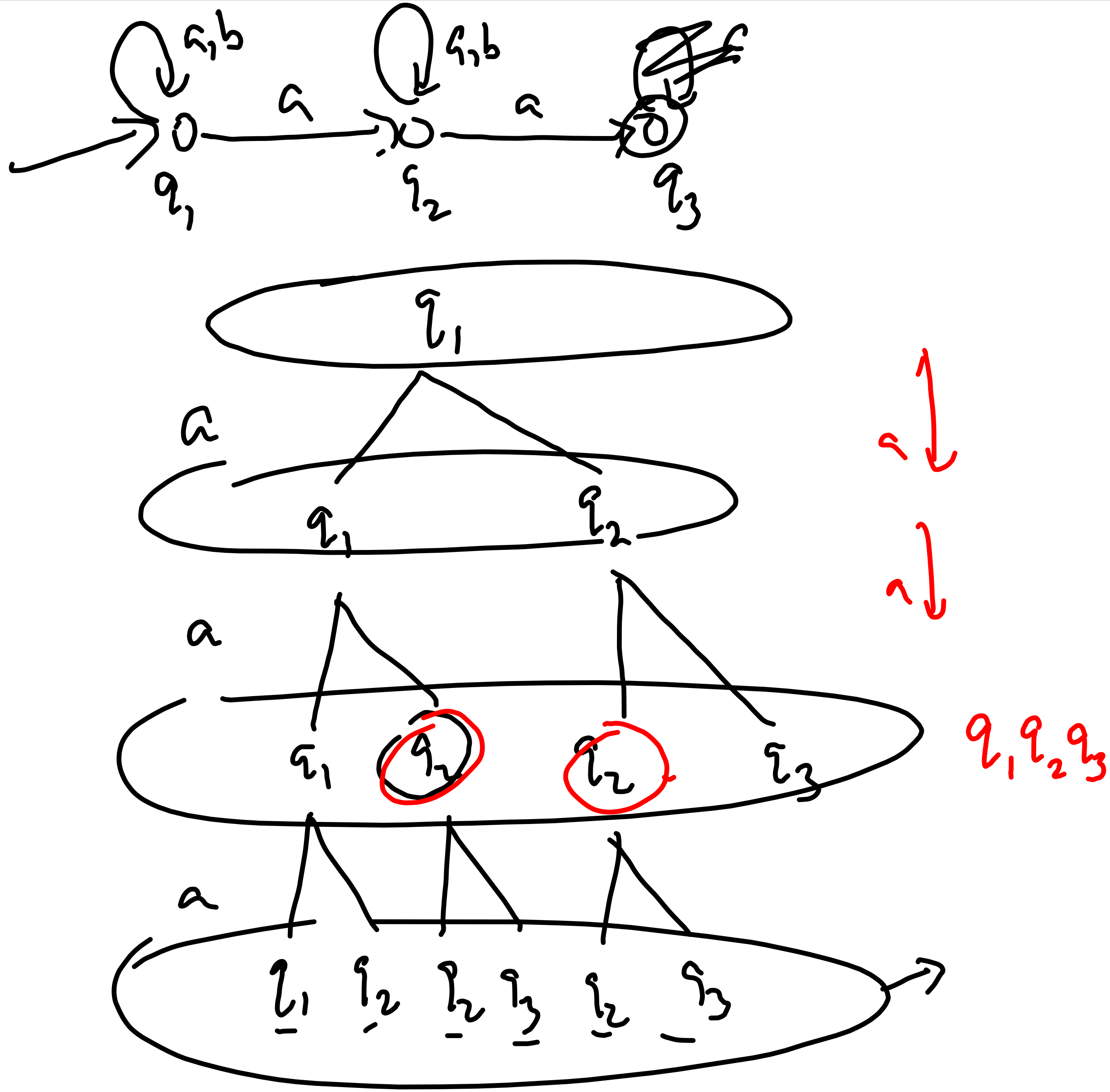


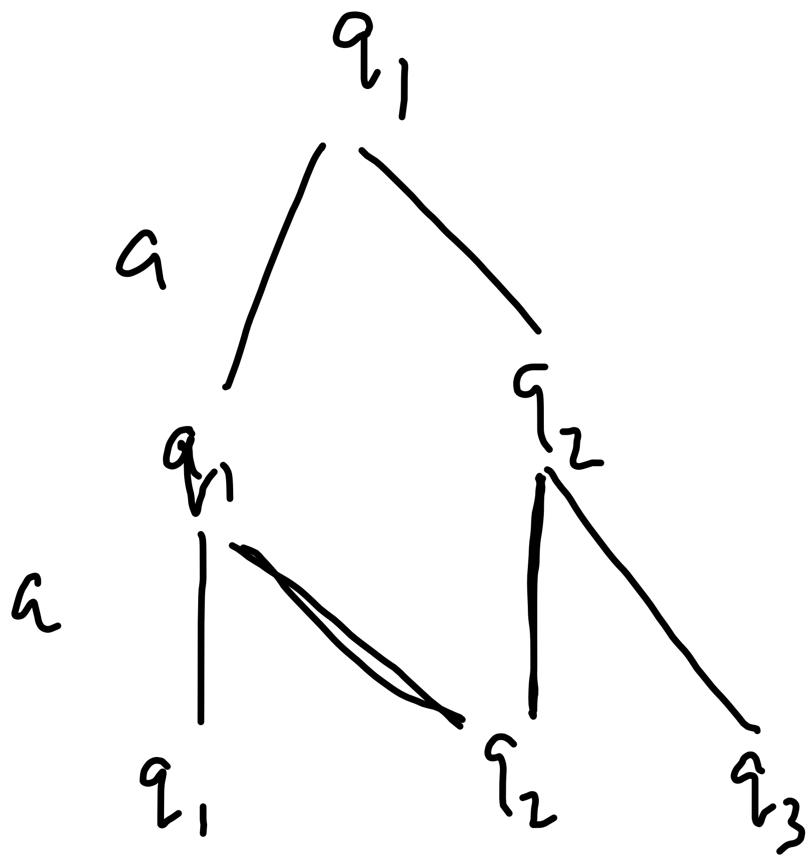
a ✓

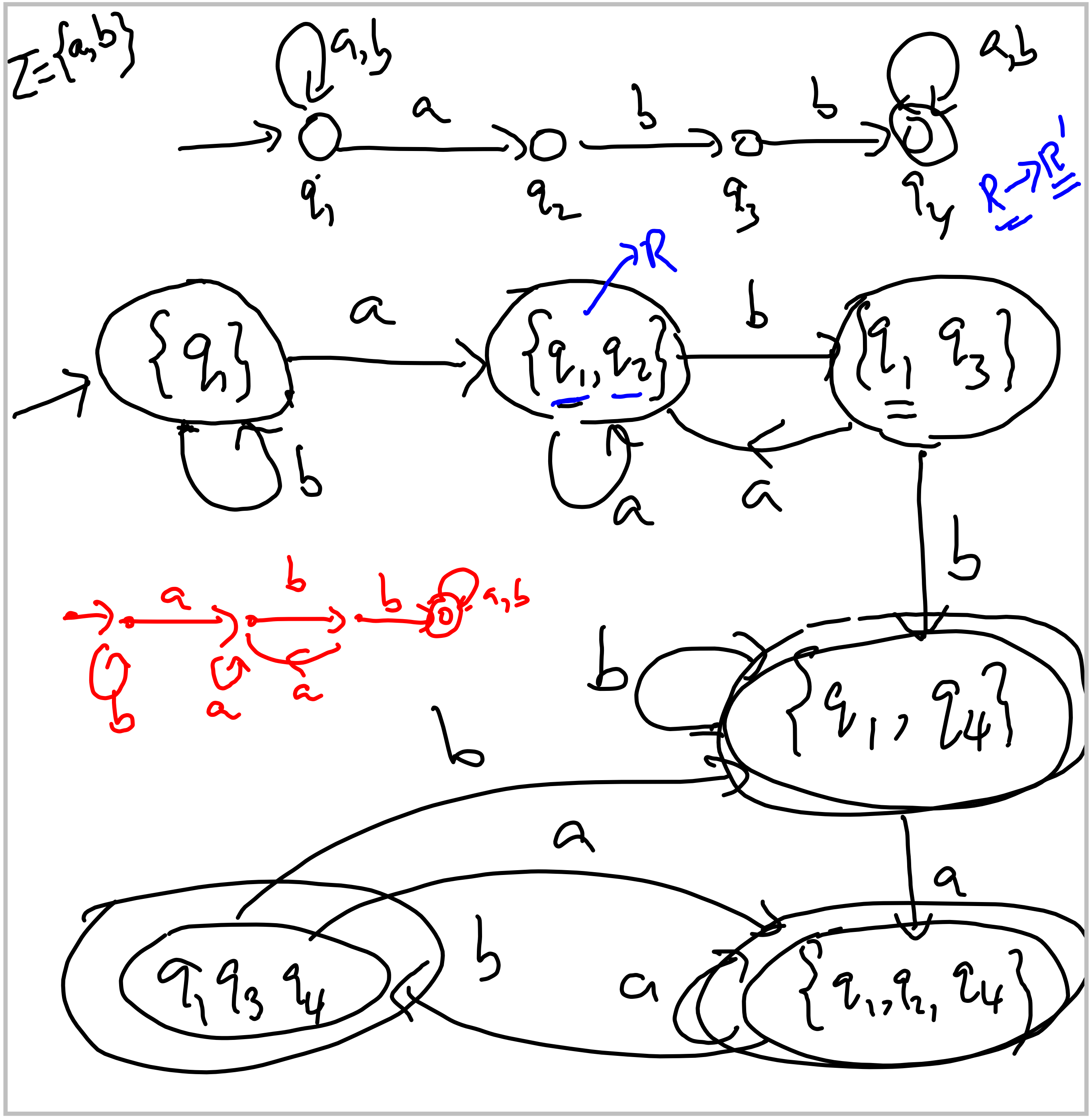
ab ✓

$L = \{ a, aa, \ldots \}$

aba,


a,b
a,b
a
a
q1
q2
q3
a
a
a
q1 q2 q3


q_1
a
q_1
q_2
a
q_1
q_2
q_3


Σ={a,b}
a,b
a
b
b
a,b
q1
q2
q3
q4
R→R'
→R
{q1}
a
{q1,q2}
b
{q1 q3}
b
a
a
b
b
b
{q1, q4}
a
a
b
{q1 q3 q4}
b
a
{q1,q2, q4}
a
b
b
a,b
b
a
a

<u>Thm</u> For every NFA A, there is a DFA B s.t. $L(A) = L(B)$

NFA $A = (Q, \Sigma, \delta, q_0, F)$

DFA $B = (Q', \Sigma, \delta', q_0', F')$

$$Q' = \mathcal{P}(Q)$$

$$q_0' = \{q_0\}$$

$$F' = \{R \subseteq Q \mid R \text{ has some accept state in it" i.e. } R \cap F \neq \emptyset\}$$

$\underline{\delta'}$

$R \subseteq Q$
$a \in \Sigma$

$$\underline{\delta'(R, a)} = \{q \in Q \mid \exists r \in R.\ q \in \delta(r, a)\}$$

$$\delta' : Q' \times \Sigma \longrightarrow Q'$$

Induction on $|w|$,

On w, if R is the exact set of states that NFA reaches on w,

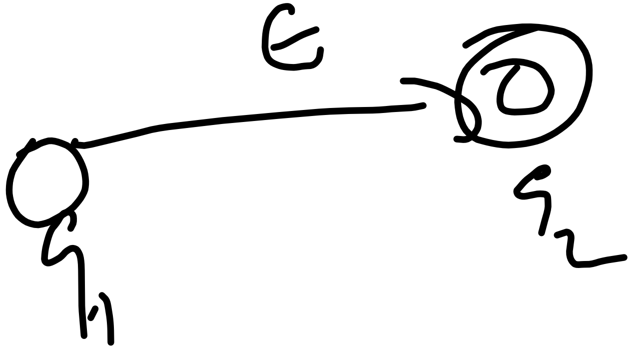
then DFA reaches R on w.

$|w| = 0$ $\qquad$ $\underline{w = \epsilon}$

$\{q_0\}$

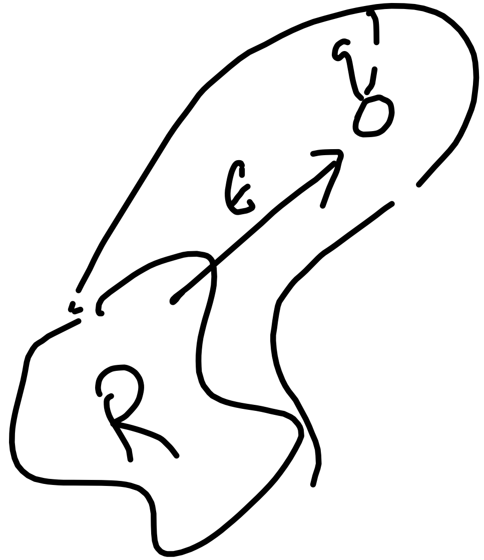
NFA $\xrightarrow{\quad w \quad} R \xrightarrow{\ a\ } R'$

DFA $\xrightarrow{\qquad} R \xrightarrow{\ a\ } R'$


ε
q1
q2


q1

~~Add~~ Handling ϵ-transitions.



$R \subseteq Q$

$E(R) = \{ q \mid q$ is reachable from some state in R using 0, 1 or more epsilon transitions $\}$

$R \subseteq E(R)$

$$\delta'(\underline{R}, \underline{a}) = \left\{ q \in Q \;\middle|\; \exists r \in R \;\; q \in E(\delta(\underline{\underline{q, a}})) \right\}$$

Initial state

$$q_0' = E(\{q_0\})$$

$$\boxed{NFA \equiv DFA}$$

Reg lang. DFAs = NFAs

DFAs : $\cup, \cap, \overline{}$

NFAs : $\cup, \cap, \circ, *$

Complement is hard

L^r

$L_1 \cup L_2$

$L_1 \cap L_2$

$L_1 \circ L_2$

L_1 L_2

L_1^*

$\overline{L_1}$

Algebraic characterization

$(a+b)^* c$

Regular expressions

$\Sigma = \{a, b, c\}$

$\{a\}, \{b\}, \{c\}$

$\{\epsilon\}, \emptyset$

$\circ$ concat

$*$ Kleene-$*$

$\cup$, $+$ Union

Close under $\circ, *, \cup$.

Expression.

$a \in \Sigma$

$$R ::= a \mid \epsilon \mid \emptyset \mid R_1 \cup R_2 \mid R^* \mid R_1 \circ R_2$$

$\boxed{a \cdot b^*}$ $(a \cdot (b^*))$ $\to \{ab^i \mid i \geq 0\}$

RE

$= \{a, ab, abb, \ldots\}$

$(a \cup b)^* \circ a$

$a = \{a\}$

$b = \{b\}$

$a \cup b = \{a, b\}$

$(a \cup b)^* = \{\epsilon, a, b, ab, ba, \ldots\}$

$L = \{w \cdot a \mid w \in \{a, b\}^*\}$

$= \{a, aa, ba, aaa, \ldots\}$

- $\{ w \mid |w| = 0 \bmod 3 \}$

$$\left[(a+b) \cdot (a+b)(a+b) \right]^*$$

- $\{ w \mid$ ~~|w|~~ $=$ w contains an 'a' $\}$

$$(a+b)^* \cdot a \cdot (a+b)^*$$

- $\{ w \mid$ w has aab as a substring $\}$

$$(a+b)^* \cdot aab(a+b)^*$$

- $\phi^* = \phi$

$$R \cup \phi = R$$

$$R \circ \epsilon = R$$

- $R \quad R'$

$$L(R) \cap L(R')$$

- $\left((b \cup c)^* \cdot a \cdot (b \cup c)^* a (b \cup c)^* \right)^*$