Exam #1

7-9 pm

Tuesday 3 October

1404 Siebel

Conflict? email Margaret ASAP

- Pumping Lemma

  Necessary but not sufficient.

- Equivalence/Congruence characterization of Reg Lang.

$L_0 = \{ a^i b^k a^k \mid i \geq 1,\ k \geq 0 \}$

$\cup\ b^* a^*$.

$\exists p.\ \forall w \in L\ \ |w| \geq p\ \ \exists x, y, z$

- $w = xyz$
- $|xy| \leq p$
- $|y| > 1$
- $\forall i\ \ xy^iz \in L$

$p = 1$ $\quad ab^ka^k$ $\quad x = \epsilon$

- $y = a$

$a^i b^k a^k \in L$ $\quad z = b^k a^k$

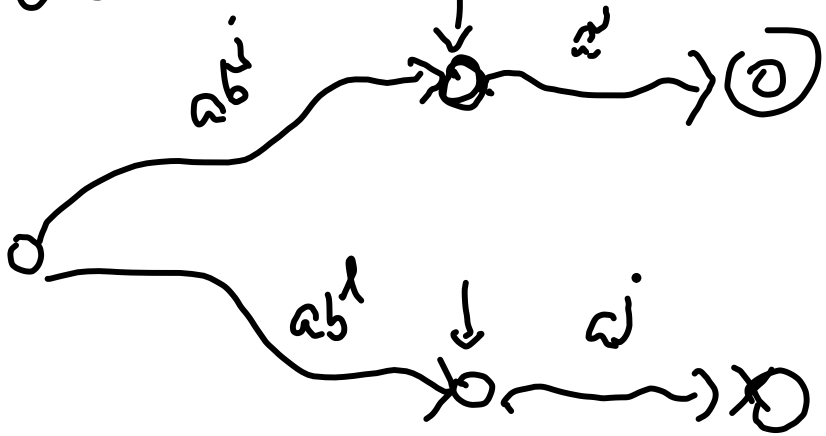
- L is reg $\Rightarrow$ PL hold.

PL holds $\not\Rightarrow$ L is reg

$L_0 = \{ a^i b^k a^k \mid i \geq 1,\ k \geq 0 \} \cup b^k a^k,$

$\{ ab^k \mid k \geq 0 \}$ — ∞ set

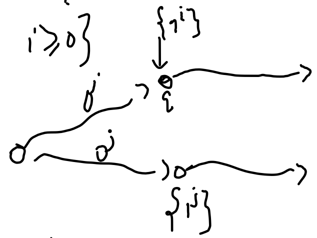
$ab^j \quad ab^l \qquad j \neq l$

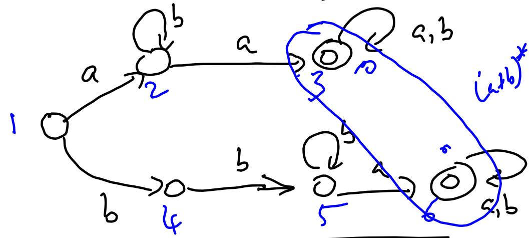


$\underline{\underline{ab^j}} \cdot \boxed{\{ a^i \}}.$

$L = \{0^n 1^n \mid n \in \mathbb{N}\}$

$\{0^i \mid i \geq 0\}$





$\rightarrow ab^*a(a+b)^*$
$+\ bbb^*a(a+b)^*$

aba 3 $(a+b)^*$
bba 6

q a $\rightarrow$ $b^*a(a+b)^*$ L_1
b $\rightarrow$ $bb^*a(a+b)^*$ L_2

In any DFA for L, after reading a, you must be in different states

A state q is "responsible" to check whether the word read from that point belongs to a regular language L_q

$$L_q = \{ w \mid q \xrightarrow{w} q_f \in F \}$$

Suppose x & y are such that

$$\{w \mid xw \in L\} \neq \{w \mid yw \in L\}$$

then x & y must lead to diff. states.

$L = a^* b a^*$

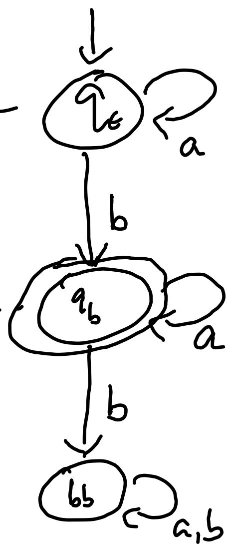
ϵ : $S_\epsilon = a^* b a^*$

a : $S_a = a^* b a^*$

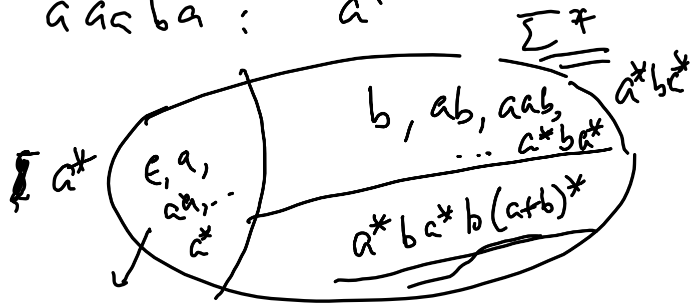
b : $S_b = a^*$

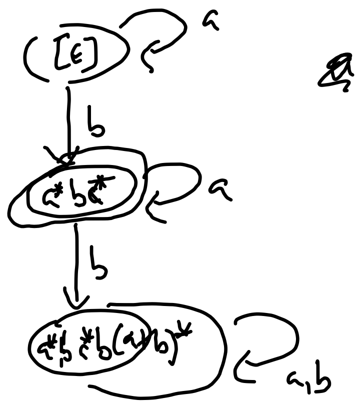
ba : $S_{ba} = a^*$

bb : $S_{bb} = \emptyset$



$aaaba$: a^*





$[\epsilon] \xrightarrow{a} [a]$

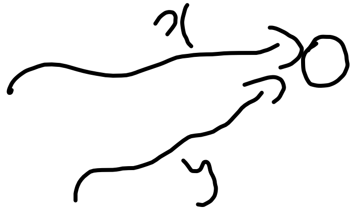
$[\epsilon] = [a]$

$[x] \xrightarrow{a} [xa]$

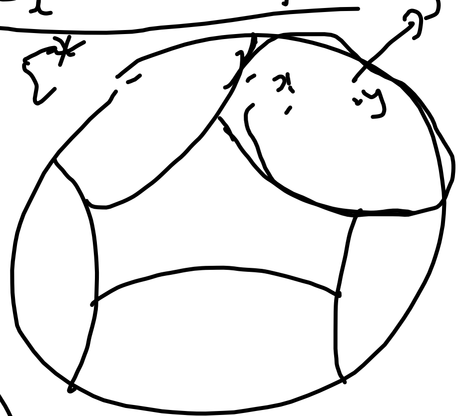
Let L be any language over Σ

$\sim_L$: $x \sim_L y \iff \forall z \in \Sigma^* . (xz \in L \iff yz \in L)$

$\iff S_x = S_y$



$\therefore$ Number of distinct S_x's is finite.



$\sim_L$

# of eq. classes is finite.

Theorem L be language over Σ.

L is regular iff $\sim_L$ has finitely many equivalence classes.

$\sim_L$ an equivalence relation:

$x \sim_L y$ iff $\forall z : (xz \in L \Leftrightarrow yz \in L)$

$x \sim_L x$ ✓

$x \sim_L y \Rightarrow y \sim_L x$

$x \sim_L y$ & $y \sim_L z$: $\underbrace{S_x = S_y = S_z}$

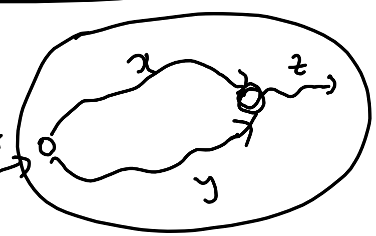
So $x \sim_L z$.

Theorem L be language over Σ.

L is regular iff $\sim_L$ has finitely many equivalence classes.

$(\Rightarrow)$ L is reg

A DFA for L



$x \approx_A y$ iff $\exists q.\ q_{in} \xrightarrow{x} q$ & $q_{in} \xrightarrow{y} q$

If $x \approx_A y$ then $x \sim_L y$

— How many $\approx_A$ equivalence classes are there?

$= |Q|$

So # of $\sim_L$ eq. classes $\leq |Q|$

$\approx_L$

$|Q|$

($\Leftarrow$) If $\sim_L$ has finitely many eq. classes, then L is regular.

If $x \sim_L y$ then $\forall z.\ \underline{xz \sim_L yz}$

$$\underline{\underline{S_x}} = \underline{\underline{S_y}}$$

$$S_{xz} = S_{yz}$$

$x \equiv y \bmod 7$

$x+1 \equiv y+1 \bmod 7$

$x \sim_L y \quad \underline{\forall z'} : \quad xz' \in L \Leftrightarrow yz' \in L$

$\underline{x\, \underline{\underline{z'z}} \in L \Leftrightarrow yz'z \in L}$

"Right-Invariant" - $\sim_L$ is RI.

If $\sim_L$ has finitely many eq classes, then L is regular.

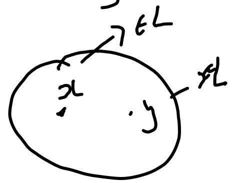
$$A = (Q, \Sigma, q_0, \delta, F)$$

$$Q = \{ [x]_{\sim_L} \mid x \in \Sigma^* \}$$

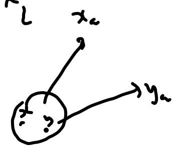
finite.

$$q_0 = [\epsilon]_{\sim_L}$$

$$F = \{ [x]_{\sim_L} \mid x \in L \}$$



δ : $[x]_{\sim_L} \xrightarrow{a} [xa]_{\sim_L}$



$x \sim_L y$

$xa \sim_L ya$

Inductively: After reading x, automaton will be in state $[x]$.

$$L = (a+b)^* a (a+b)^* a (a+b)^*$$

$[\epsilon] : L$

$[a] : (a+b)^* \cdot a (a+b)^*$

$[b] : L$

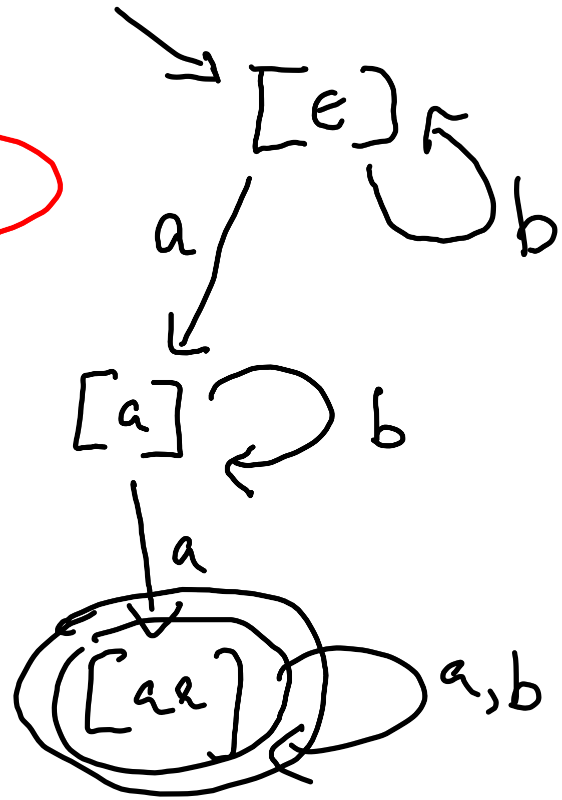
$ba \sim [a]$

$ab \sim$

$[aa] : (a+b)^*$

$[aaa] : (a+b)^*$

$[aab] :$



L is reg iff $\sim_L$ has finitely many eq classes.

Myhill Nerode

$$L \longrightarrow \sim_L \longrightarrow A$$

NOT in Textbook

minimal automaton