Exam is graded!

50's	26
40's	24
30's	22
20's	9

max 58

Reductions

Know H is undecidable

often A_{TM} or E_{TM}

Reduce H to E

Therefore E must be undecidable

Assume TM that decides E
Build TM that decides H

$$EQ_{TM} = \{ \langle M_1, M_2 \rangle : L(M_1) = L(M_2) \}$$

is undecidable

Proof: Reduction from E_{TM}

Assume TM R that decides EQ_{TM}

Let $T_\emptyset$ be some TM s.t. $L(T_\emptyset) = \emptyset$

Define TM S deciding E_{TM} as follows

- input $\langle M \rangle$
- return $R(\langle M \rangle, \langle T_\emptyset \rangle)$

$HALTEMPTY_{TM}$

$= \{ \langle M \rangle : M$ halts on blank input $\}$

is undecidable

Proof: Reduction from $HALT_{TM}$

Suppose R decides $HALTEMPTY_{TM}$

Define S that decides $HALT_{TM}$ as follows

S: • input $\langle M, w \rangle$

• construct new TM M_w with w hardcoded into M

M_w: ignores its input
return $M(w)$

• return $R(\langle M_w \rangle)$

$Regular_{TM} =$

$\{ \langle M \rangle : L(M) \text{ is regular} \}$

is undecidable.

Proof: Reduction from A_{TM}

Suppose R is a TM deciding $Regular_{TM}$

Need to build S deciding A_{TM}

S: • input is $\langle M, w \rangle$

• construct M'_w as follows

M'_w : input x

if x has form $0^n 1^n$

accept

else return $M(w)$

$$L(M'_w) = \begin{cases} \Sigma^* & \text{if } M \text{ accepts } w \\ 0^n 1^n & \text{if } M \text{ doesn't accept } w \end{cases}$$

• return $R(\langle M'_w \rangle)$

Rice's Theorem

If P is a set of TM s.t.

1) P is "nontrivial"

   i.e., $P \neq \emptyset$ and P doesn't contain all TMs

2) Membership in P depends only on a TM's language

   if $L(M_1) = L(M_2)$

   Then $M_1 \in P \Longleftrightarrow M_2 \in P$

Then P is undecidable

Proof: Suppose P is a set meeting conditions above. Suppose P is decidable. Let TM N decide P

Suppose WLOG that $T_\emptyset \notin P$
where $T_\emptyset$ is a TM s.t. $L(T_\emptyset) = \emptyset$

Let $T \in P$ $\Longleftarrow$ P non-trivial

Construct TM S deciding A_{TM}

S: • input $\langle M, w \rangle$

• construct new TM M'_w

M': • input x

• simulate M on w

if M rejects, M' rejects

if M accepts w

then return $T(x)$

• return $N(\langle M' \rangle)$

Title: Nov 16-1:07 PM (8 of 13)

$L(M') = \emptyset$ if M doesn't accept w

or $L(T)$ if M accepts w

if $L(M') = \emptyset$ then $L(M') = L(T_\emptyset)$

since $T_\emptyset \notin P$, then $L(M') \notin P$

uses property 2 of Rice's Theorem

so $N(\langle M' \rangle)$ reject

if $L(M') = L(T)$

then since we know $T \in P$

we know that $M' \in P$

so $N(\langle M' \rangle)$ accept

TM problems

- properties of language $\Rightarrow$ undecidable
- properties of TM's structure

  e.g. TM has ≤ 13 states

  $\Rightarrow$ decidable

- properties of TM behavior

  eg. does TM ever move left on blank input?

  $\Rightarrow$???

$L_{111} = \{ \langle M \rangle :$ M prints 3 one's in row on blank input $\}$

is undecidable

Idea: Given input M

Build M' s.t.

M' doesn't print 1's while it computes

& M' prints 3 ones when it accepts

Let R decide L_{111}.

Construct S deciding ~~ATM~~ $HALT_{TM}$

S: • input <M, w>

• Construct new TM M'_w as follows

M'_w: • ignores input

• hardcode w into M'_w

• run code for M on w <u>except</u> 2 mods to M's code

• wherever M uses 1, replace it with 1 = a new symbol not in M'_w's alphabet

• when M would halt, M'_w first prints 3 one and then halts

Final step in S: $R(\langle M'_w \rangle)$

Title: Nov 16-1:29 PM (12 of 13)

$L_{left} = \{ \langle M \rangle :$ M ever moves left on blank input $\}$

is decidable

Given TM M

Simulate M on blank input

Watch for left move

After $|Q|$ moves, it must be looping

So we can quit watching after $|Q|+1$ steps.