Non-Contextfree Languages

& Properties of CFLs.

$$L = \{ a^n b^n c^n \mid n \geq 0 \}$$

is not context free

$$L' = \{ a^n b^n \mid n \geq 0 \}$$

is a CFL

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

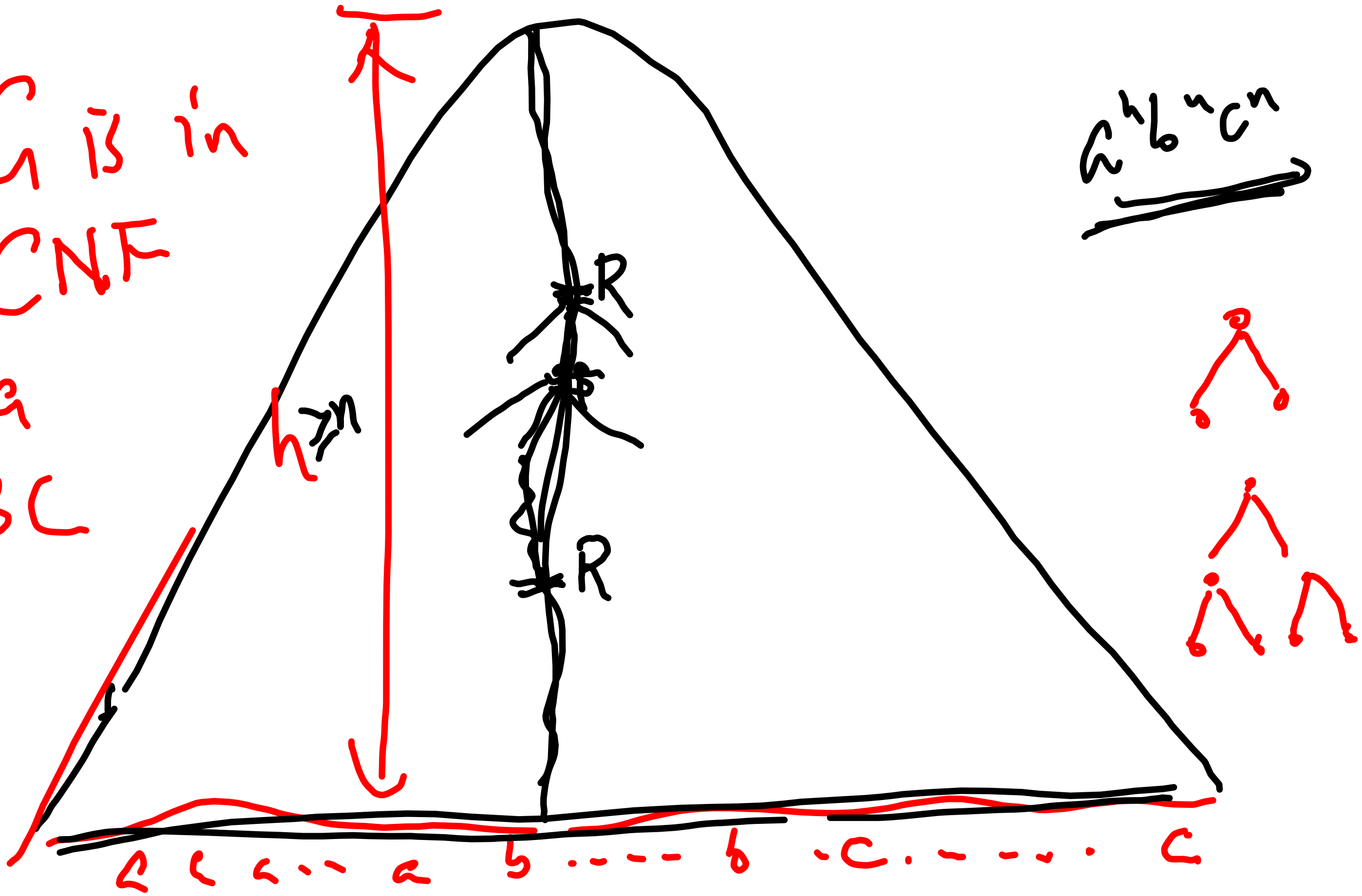
Assume L is a CFL.

Then there is a CFG G s.t. $L(G) = L$.

Assume G is in CNF

$A \to a$

$A \to BC$



Let G have n variables.

Consider the parse-tree for

$$a^{2^{n+1}} b^{2^{n+1}} c^{2^{n+1}}$$

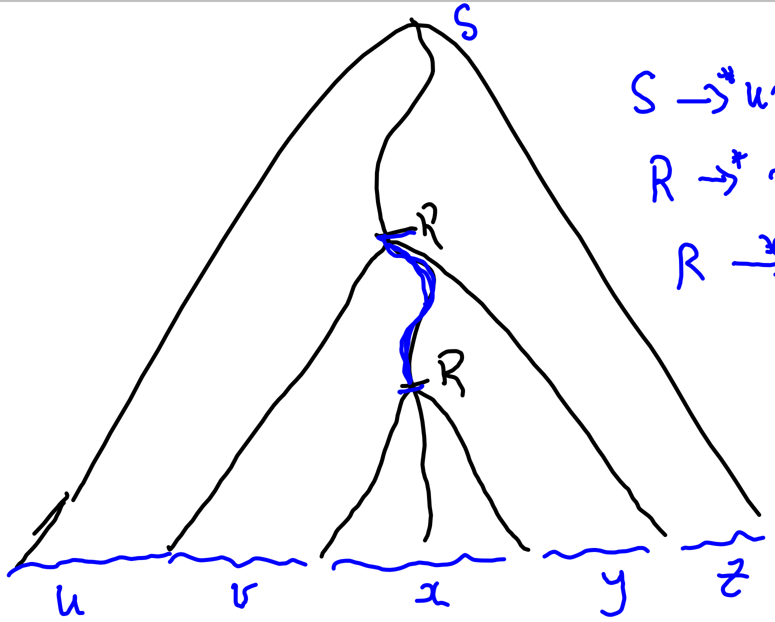
So Height of parse tree must be at least $n+1$

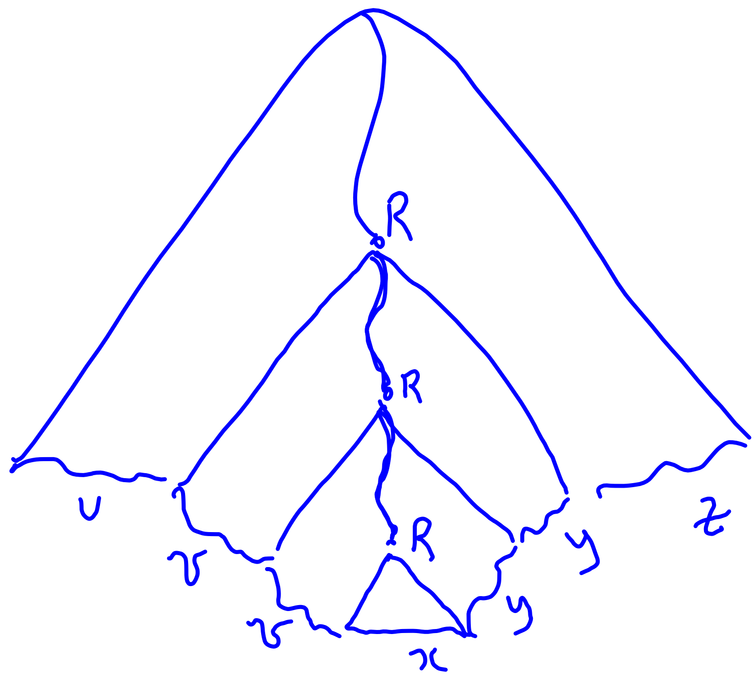
So some variable R ~~repeats~~ occurs twice on some path in the tree

$S \xrightarrow{*} uvxyz$

$R \xrightarrow{*} vxy$

$R \xrightarrow{*} x$





$\exists\ u\ v\ x\ y\ z = a^n b^n c^n$ $\qquad n = 2^{|V|+1}$

$u\underline{\underline{v}}^2 x \underline{\underline{y}}^2 z \in L(G)$

Case 1 $\quad v$ has only one ~~letter~~ character & y has only one character

$uv^2xy^2z \notin L$

Case 2 $\quad v$ or y has more than one character

$uv^2xy^2z \notin a^*b^*c^*$

$\notin L$

Pumping Lemma

If L is a CFL,

$\exists\, p \geq 0$

s.t. $\forall\, s \in L$, $|s| \geq p$

$\exists\, u, v, x, y, z$ s.t.

$s = \underline{uvxyz}$

and

1) ~~ex~~ $uv^ixy^iz \in L$

$\forall i \geq 0$

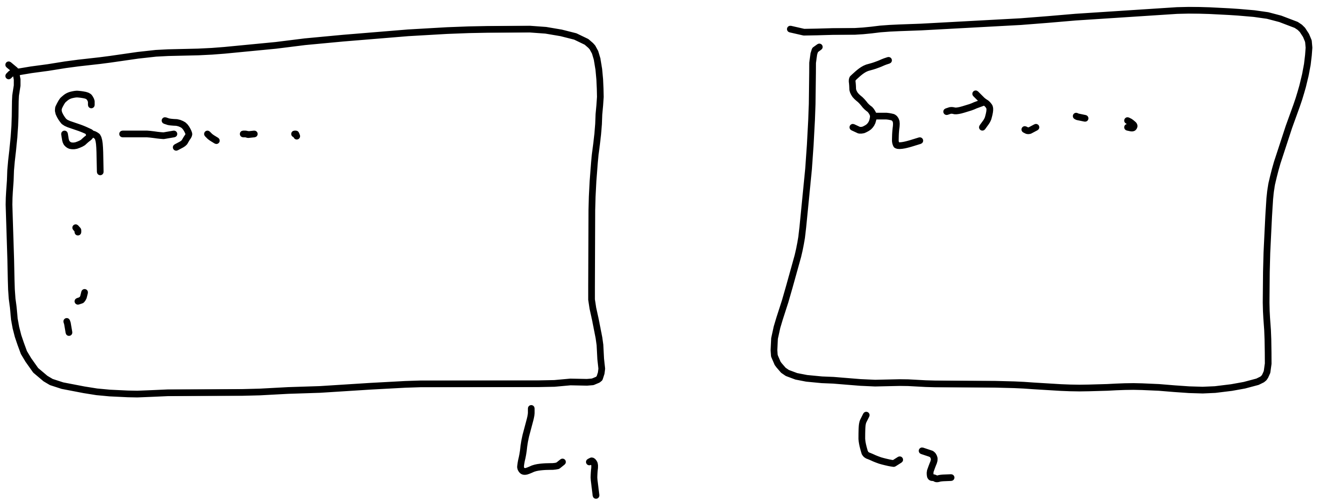
2) $|vy| \geq 1$

3) $|\underline{vxy}| \leq p$

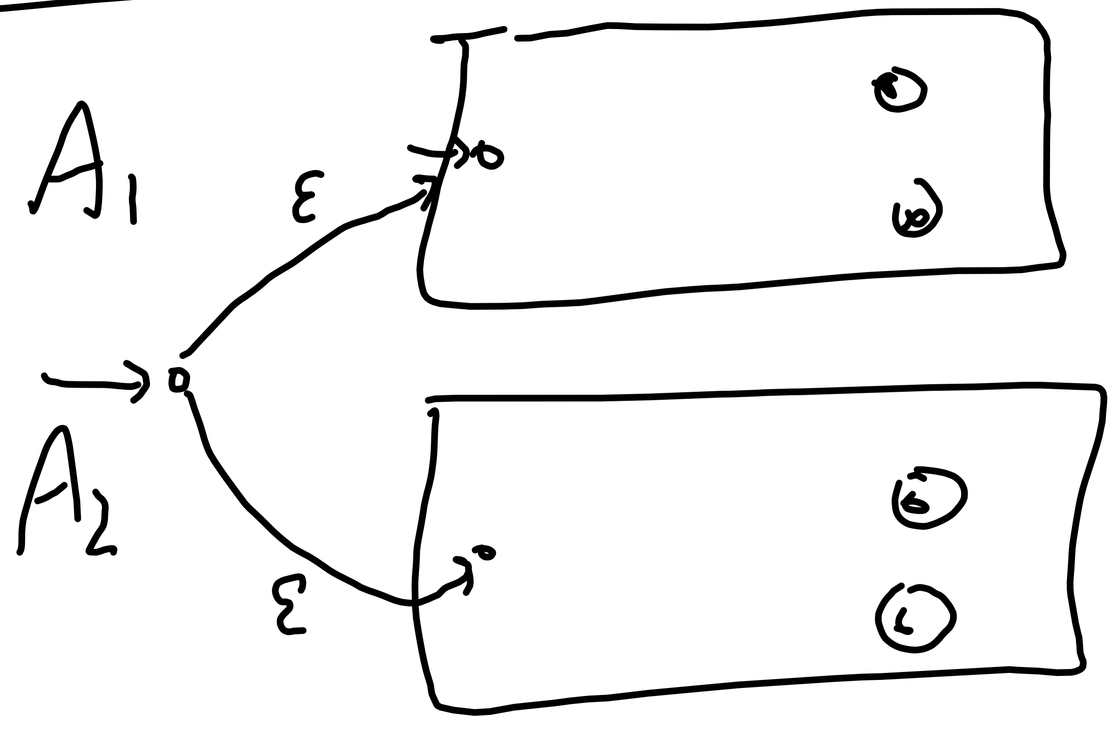
Closure properties for CFLs

Closure under union ✓ Yes

$$S \to S_1 \mid S_2$$

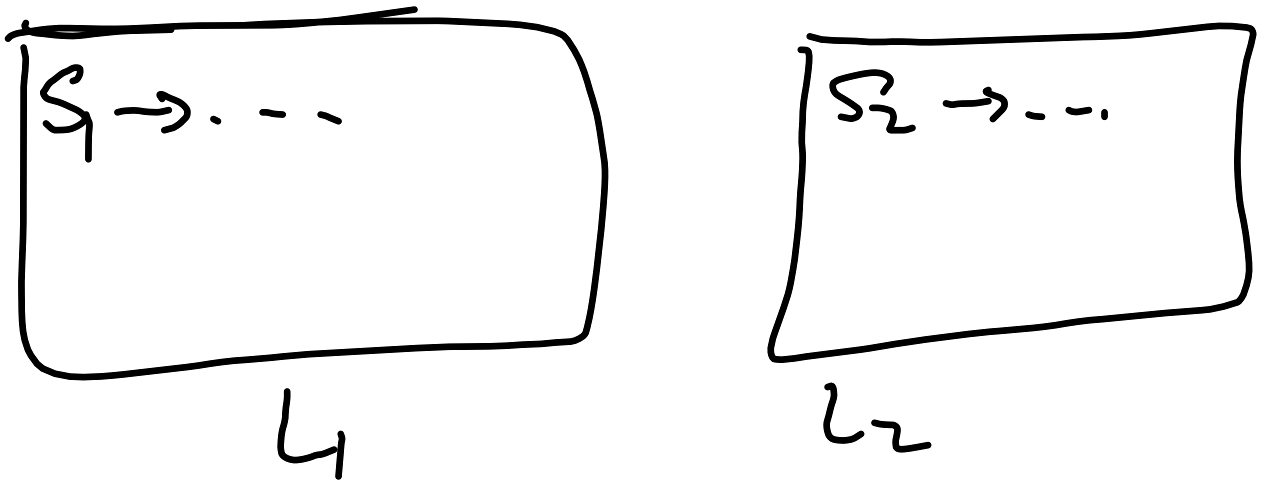


Union using PDA



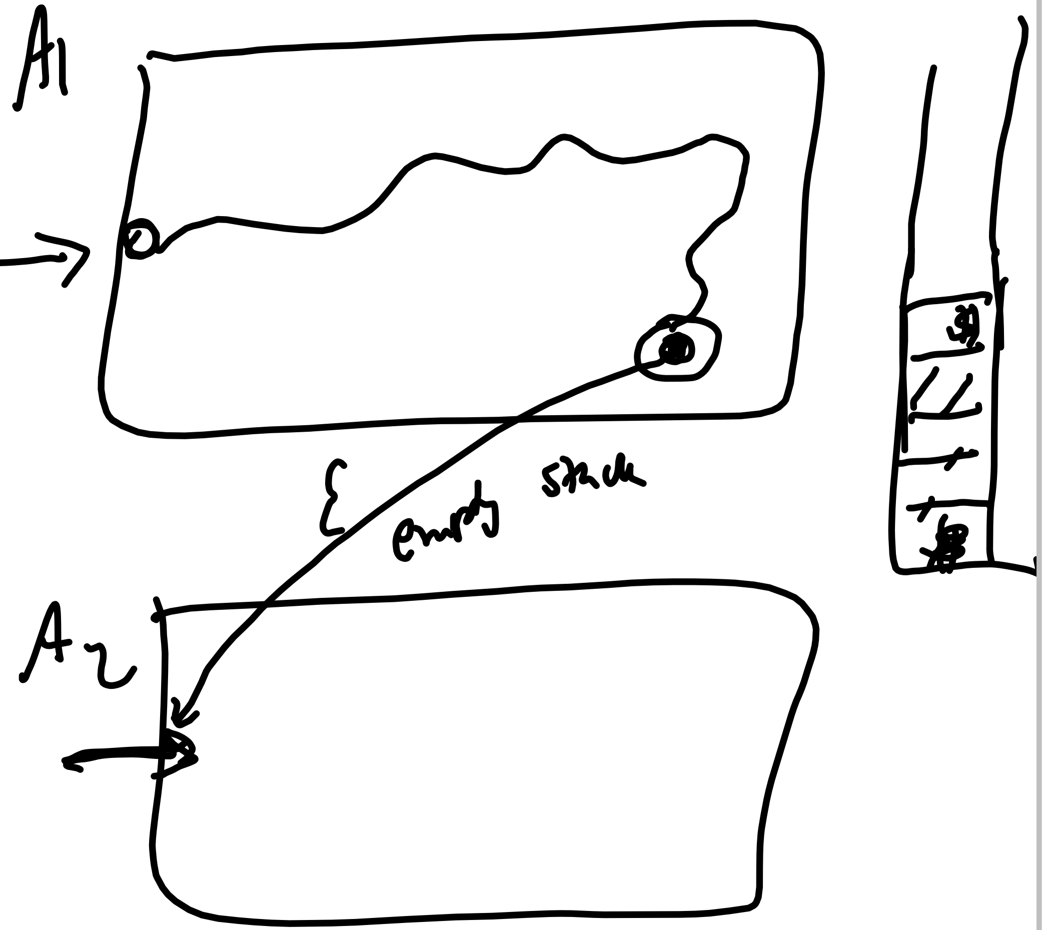
Closure under concatenation

$$S \rightarrow S_1 S_2$$



PDA : closure under concatenation

$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \{\$\} \longrightarrow$



Closure under Kleene-* Yes

L_1 - CFL

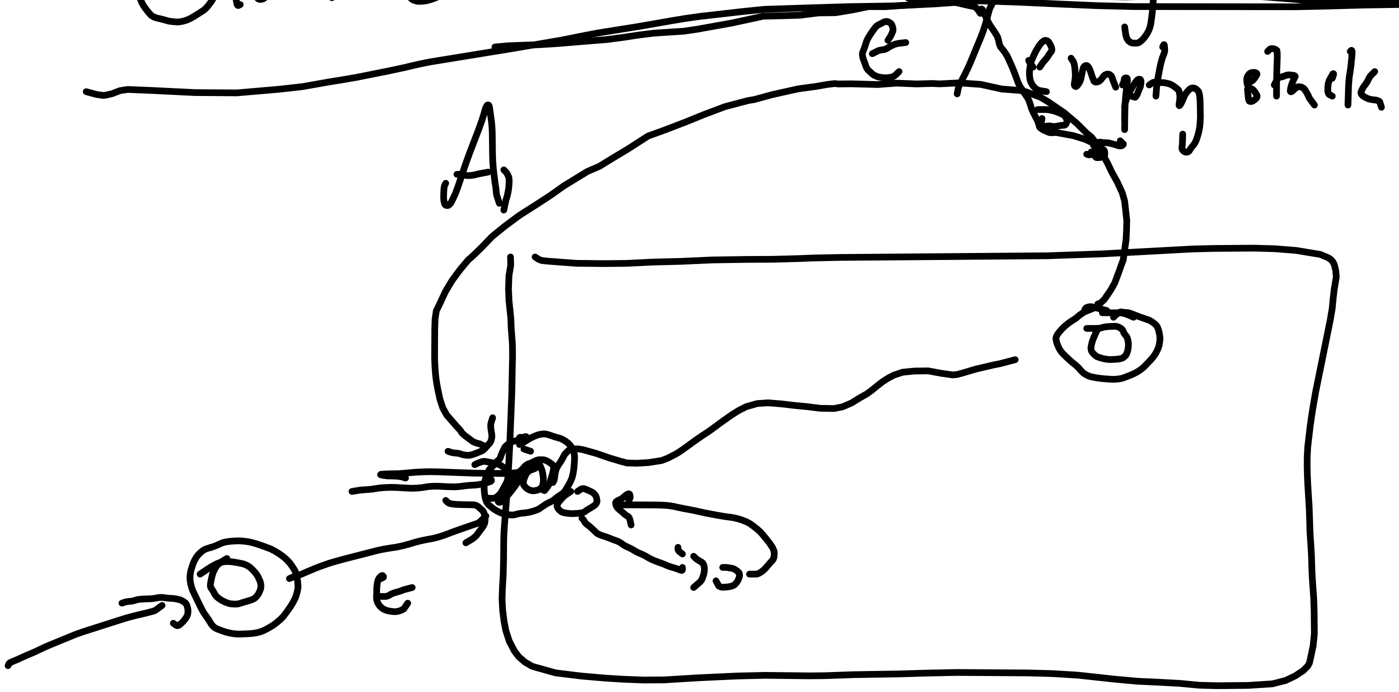
L_1^* - CFL?

$$S \to \epsilon \mid S_1 S$$

$S_1 \to \ldots$

L_1

Closure under * using PDAs?



Closure under intersection? No!

$L = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL

$L_1 = \{a^n b^n c^i \mid n \geq 0, i \geq 0\}$

$L_2 = \{a^i \underline{b^n c^n} \mid n \geq 0, i \geq 0\}$

$L = L_1 \cap L_2$

$\underline{L_1}$

$S \longrightarrow S_1 T$

$S_1 \longrightarrow \epsilon \mid a S_1 b$

$T \longrightarrow \epsilon \mid c T$

Both L_1 & L_2 are CFLs

but $L_1 \cap L_2$ is not

so CFLs are not closed under intersection

Closure under complementation? No!

$L = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL

$\overline{L} = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\} \cup \overline{\{a^* b^* c^*\}}$

$\overline{L}$ is a CFL.

PDA for $\overline{L}$:

- nondet guesses to check $i \neq j$ or $j \neq k$
  - For $i \neq j$, it pushes d on reading a's, pops d on reading b's, and hence checks $i \neq j$
  - For $j \neq k$, it pushes d on reading b's, pops d on reading c's, and checks $j \neq k$

$\overline{L}$ is a CFL but $\overline{(\overline{L})} = L$ is not a CFL

So CFLs are not closed under complement.

Closure under homomorphisms

$$h : \Sigma \longrightarrow \Pi^*$$

h : $a \longrightarrow 0$
$b \longrightarrow 1$
$c \longrightarrow 00$

$h(ca) = 000$

L is a CFL over Σ,
Is $h(L)$ a CFL where
h is a homomorphism
$h : \Sigma \longrightarrow \Pi^*$

$S \to S_1 S_2 \mid \ldots$

$T \to aT'$

$h(a) = bc$

$T \to bcT'$

Context free languages are closed under homomorphisms.

DPDA $\cancel{\subseteq}$ $\subsetneq$ NPDA

Claim

No DPDA can accept $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\} \cup \overline{a^* b^* c^*}$

Proof

Assume a DPDA A accepts this language.

$\bar{L}$ is accepted by DPDA

$\bar{L} = \{a^n b^n c^n \mid n \geq 0\}$

Contradiction.

Other non-CFLs $\quad |\Sigma| \geq 2$

$$L = \{ ww \mid w \in \Sigma^* \}$$

is not a CFL.

Recall $L = \{ ww^r \mid w \in \Sigma^* \}$ is a CFL

PDA A

$L(A) \neq \emptyset$

Decidable

	RL	CFL
Clos. under: ∪, ∘, *, homomorphisms	✓	✓
Closure under complement & ∩	✓	X
Deciding Emptiness	✓	✓
Deciding Inclusion Equivalence	✓	X

Title: Oct 12-1:28 PM (23 of 29)

$L_1 \qquad L_2$

Is $L_1 \subseteq L_2$

$L_1 \subseteq L_2 \iff \underline{L_1 \cap \overline{L_2} \neq \emptyset}$

Reg lang $\subseteq$ CFL ?

is <u>undecidable</u>

Applications of CFLs

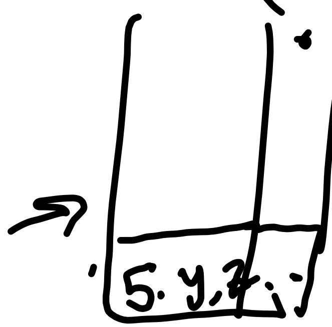
- Formal grammars / parsing
- Natural language processing
-

Verification / Static Analysis of Programs

```
foo (int x)
{
  :
  y := foo(x-1)
}
```

5

$y = z/x$



Compression of text

ababc . d ababc . e . .

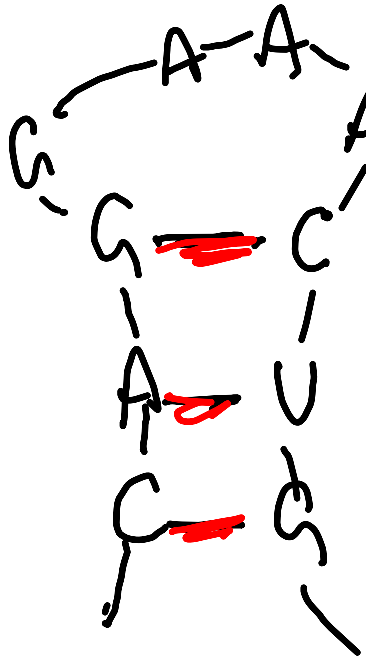
CFG

$S \rightarrow TdTe$

$T \rightarrow ababc$

RNA structure analysis

CAGGA ACUG



RNAs

using

CFGs

→ CSG