Model solution for 5a & 5b.

Note Title 10/12/2006

5 (a) We assume $L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

We must show $L' = \{a^n \# (bc)^n \mid n \geq 0\}$ is not regular.

Proof by contradiction:

Assume L' is regular.

Define a homomorphism

$$h : \{a, b, c, \#\} \rightarrow \{0,1\}^*$$

as:

$$h(a) = 0$$
$$h(\#) = \epsilon$$
$$h(b) = \epsilon$$
$$h(c) = 1$$

For any $n \geq 0$, $h(a^n \# (bc)^n) = 0^n \cdot \varepsilon \cdot (\varepsilon \cdot 1)^n$
$= 0^n 1^n$

So $h(L') = \{0^n 1^n \mid n \geq 0\} = L$

If L' was regular, then $h(L') = L$ will be regular as well, which we know is not the case. Hence L' is not regular.

<u>5(b)</u>

L is the set of all words over $\{a, b\}$ that have an equal number of a's & b's.

i.e. $L = \{w \mid \#\text{ of a's in } w = \#\text{ of b's in } w\}$

Turn Over

It is easy to argue that all words in L have an equal number of a's & b's.

Proof by induction on # of rules used.

Initially we have the empty string (by Rule 1) which has 0 a's & 0 b's.

If Rule 2 is used:

if x, y have equal # of a's & b's

then so do $axby$ & $bxay$.

So all words in L have equal number of a's & b's

This is all that you need to write since the question doesn't ask for a formal proof.

We can also show that L contains all words that have an equal # of a's and b's.

Proof by induction on the length of words.

Turn Over.

$\underline{n=0}$: The empty word belongs to L.

$\underline{n>0}$. <u>Inductive hypothesis</u>

Assume that for all strings w', $|w'|<n$, if w' has an equal number of a's and b's, then $w' \in L$.

Let $|w|=n$ and let w have an equal number of a's & b's.

<u>Case 1</u>. ~~If~~ <u>w starts with an 'a'</u>, :

$w = a w'$

Let $w = a w_1 b w_2$ where

this b is the first time ~~to~~ along the word that the number of a's become equal to number of b's.

i.e. $\forall$ prefix u of w_1, au has more a's than b's.

Note that such a split must be there since w has equal # of a's and b's.

Then w_1 has an equal # of a's & b's

and w_2 has an equal # of a's & b's.

So by the induction hypothesis, w_1 & w_2 belong to L.

Hence by Rule(2), $a w_1 b w_2 \in L$.

<u>Case 2</u> <u>w begins with b</u>

This is same as Case 1 except that we use the "bxay" rule of Rule(2) to show $w \in L$.

$\square$