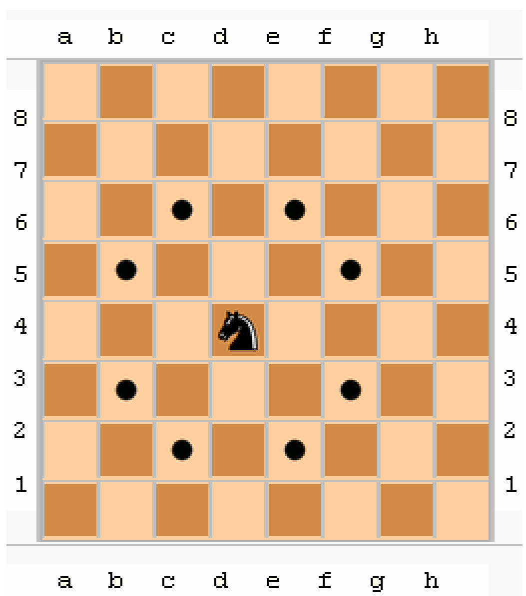
CS173 Discrete Mathematical Structures
Fall 2006
Homework #7
due Sunday, October 22, 2006, 8:00 a.m.

1) Use induction to prove that
   a. $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2^n-1}+\frac{1}{2^n}\geq 1+\frac{n}{2}$ for n>0
   b. $2^n < n!$ for n>3
   c. $\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$ For any group of sets: $A_1, A_2, \ldots, A_n$ where n>1.
   d. 21 divides $4^{n+1}+5^{2n-1}$, for all positive integers n>0.

2) A knight on a chessboard can move in L-shape. That is, it can move one space horizontally (in either direction) and two vertically at the same time, or one space vertically and two horizontally at the same time. The figure below illustrates these rules:



Assume that you have a square board that's larger than 3x3. Prove that if you start in the lower left corner, (0,0), that you can get to any spot (i,j) on the board in a finite number of steps. (Hint: use induction on the width of the square board)

3) Give a recursive definition of the following sequences:

a. $a_n = 2n$
b. $a_n = 2^n$
c. $a_n = 2n-1/2$
d. $a_n = n^2$
e. $a_n = 2^n+(-2)^n$

4) Part1: Give a recursive definitions for:

a. S(n) – sum of the first n positive integers
b. S(n,m) – sum of an integer n and nonnegative integer m.
c. P(n,m) – product of the integers n,m. Note that both n and m can be negative!
d. GCD(n,m) – greatest common devisor between n and m. (It's le largest number that divides both n and m).

Part2: For each of the definitions above, use pseudocode to write a recursive function that evaluates them.

5) Recall that the recursive definitions for the functions min and max are:

$\min(a_1,\ldots,a_n)=$

$\min(a_1,\min(a_2,\ldots,a_n))$ for $n>2$
$\min(a,b)=b$ if $a\geq b$
$\min(a,b)=a$ if $a<b$

$\max(a_1,\ldots,a_n)=$

$\max(a_1, \max(a_2,\ldots,a_n))$ for $n>2$
$\max(a,b)=a$ if $a\geq b$
$\max(a,b)=b$ if $a<b$

Using these definitions, prove that:

$$\min(a_1+b_1, a_2+b_2,\ldots, a_n+b_n) \geq \min(a_1, a_2,\ldots,a_n)+ \min(b_1, b_2,\ldots,b_n).$$