

CS173 Discrete Mathematical Structures  
Fall 2006  
Homework #1  
due Sunday, September 3, 2006, 8:00 a.m.

1. Post your name to the class newsgroup under the “Problem #1” thread. Include in this post the last movie you watched, and rate it from 1 to 5 stars.
2. Determine whether each of the following is a tautology, a contradiction, or a contingency. Prove part 2d by showing that the proposition is logically equivalent to TRUE.
  - a.  $(p \wedge (q \rightarrow p)) \rightarrow q$
  - b.  $p \rightarrow (q \rightarrow p)$
  - c.  $\neg((p \oplus q) \rightarrow (p \vee q))$
  - d.  $(p \wedge (p \rightarrow q)) \rightarrow q$
3. Let  $p$ ,  $q$ , and  $r$  be the propositions  
     $p$ : You have the flu.  
     $q$ : You miss the final examination.  
     $r$ : You pass the course.  
Express each of the following propositions as an English sentence.
  - a.  $q \rightarrow \neg r$
  - b.  $r \leftrightarrow \neg q$
  - c.  $(p \wedge q) \vee (\neg q \wedge r)$
4. A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning using a truth table.
5. Let  $L(x,y)$  be the statement “ $x$  loves  $y$ ,” where the universe of discourse for both  $x$  and  $y$  is the set of all people in the world. Use quantifiers ( $\forall$ ,  $\exists$ ) to express each of the following statements:
  - a. Everybody loves Raymond.
  - b. Everybody loves somebody.
  - c. There is somebody whom everybody loves.
  - d. There is somebody whom no one loves.
  - e. Nobody loves everybody.
  - f. Everybody loves himself.
  - g. Everybody loves everybody.
  - h. Someone loves at least two people.
  - i. Someone loves exactly two people.