CS 598mp: Fall 2005: Homework #3

Due on Fri 14 Oct

Hand over in class or to Colin Robertson at 3229 SC

Problem 1. (LTL $\rightarrow$ FO)

Consider LTL over infinite words, over the set of proposition $\mathcal{P}$. Let $\Sigma = 2^{\mathcal{P}}$.

For any LTL formula φ over $\mathcal{P}$, show that there is a formula $\psi(x)$ (over exactly one free variable x) in *first-order logic over* Σ*-labelled* ω*-words* such that:

- for any word $\alpha \in \Sigma^\omega$ and $i \in \mathbb{N}$,

  $$\alpha, i \models \varphi \textit{ iff } \alpha \models_I \psi(x)$$

  where $I(x) = i$.

Problem 2. (Interpretations)

Let $M_1 = (U_1, R_1, \ldots, R_k, =)$ and $M_2 = (U_2, S_1, \ldots, S_l, =)$ be two relational models with equality, where each R_i and each S_i are binary relations.
Consider an *interpretation* of M_2 in M_1, $\langle f, \varphi_f, \psi_1, \ldots, \psi_l \rangle$ given as follows:

- $f : U_2 \longrightarrow U_1$ is an injective map
- $\varphi_f(x)$ is an MSO formula over M_1 with free variable x which evaluates to true only for elements in U_1 which are in the range of f (i.e. only for elements in U_1 such that there is an element in U_2 which gets mapped to it by f).
- Each $\psi_i(x, y)$ is an MSO formula over M_1 such that for any interpretation of x and y over elements in the range of f, the formula evaluates to true if and only if the corresponding elements in U_2 satisfies S_i.
  In other words, if $v_1, v_2 \in U_2$, $f(v_1) = u_1$ and $f(v_2) = u_2$, then $\langle u_1, u_2 \rangle$ satisfies ψ_i iff $\langle v_1, v_2 \rangle$ satisfies S_i.

Show that there is an effective translation h that maps MSO sentences over M_2 to MSO sentences over M_1 such that for any MSO sentence φ over M_2, $M_2 \models \varphi$ if and only if $M_1 \models h(\varphi)$.
[Hence, if M_1 has a decidable MSO theory, then so will M_2.]