

CS 598mp: Fall 2005: Homework I
Due on Fri 9 Sep
Hand over in class or to Colin Robertson at 3229 SC

Problem 1.

Write down the following properties in MSO:

- Over the class of graphs, write the property $\varphi(x, y)$ that says “There is a path from x to y ”.
- Write the property ψ over connected graphs: $\psi(X)$ that says “ X is a cut of the graph”; i.e. removing vertices X will result in a disconnected graph.

Problem 2.

The natural way to model a graph $G = (V, E)$ is as a structure (V, E) where V is the universe and E is a binary relation. However, there is another way to model graphs: as a structure $(V \cup E, inc)$, where vertices and edges are considered to be elements of the universe, and inc is a binary relation capturing the incidence relation: $inc(e, u)$ is true iff e is an edge, u is a vertex and e is incident on u . (Note that you can check if x is an edge by checking if $inc(x, y)$ is true for some y in the universe).

Logics on this structure are (potentially) more powerful as they can quantify over *edges*. Using this structure, write down the following property:

- A formula $\varphi(x, y)$ that says “There is a hamiltonian path from vertex x to vertex y in the graph”; [A Hamiltonian path is a path from x to y that visits each vertex exactly once.] Write this in MSO.

Problem 3.

Let $\Sigma = \{a, b, c\}$.

Give a step-by-step construction of the various automata for the subformulas, and construct the automaton for the following MSO sentence:

$$\forall x(Q_a(x) \Rightarrow (\exists y.Q_b(y)))$$

You may minimize any automaton after constructing it.

Problem 4.

Fix $\Sigma = \{a, b, \$\}$.

Let L be the language of all words over Σ of the form $w\$w'$ where $w, w' \in \{a, b\}^*$ such that every three-letter contiguous subword of w also occurs as a three-letter contiguous subword in w' .

Show that L is regular. (Be smart; don't use automata!)