1. **Expectation:**

   (a) What is the expected value of the roll of a six-sided die? If you roll two dice and add the scores on the top faces, what is the expected value of the result?

   (b) On average, which gives you a higher score: $4\mathcal{D}8$, or $2\mathcal{D}10 + 6$? Here, $x\mathcal{D}y$ means rolling a y-sided dice x times, and adding the scores on the top faces. If you had to make a 19 or 20, would you rather roll a $1\mathcal{D}20$, or $2\mathcal{D}10$? If you had to make at least a 4?

2. **The Birthday Paradox:** What is the probability that 4 people in a room were all born on different days of the week? How many people do you need to gather into a room so that the probability that at least one pair of them share a birthday is at least $1/2$?

3. **Majority-Trees:** Consider a perfect ternary tree, of height h. The root has 3 children, each of which has 3 children, and so on until the leaves, at depth h. Let $n = 3^h$ be the number of leaves in the tree, and suppose each leaf is labeled with either a 0 or a 1. To *evaluate* a node, you must find its label, which (for non-leaf nodes) is defined as the label of the majority of its children(For example, if the children of a node has values 1,0,0 respectively, then the node has a value of 0); we evaluate a tree by finding the label of its root.

   Show that any deterministic algorithm looks at all the leaves in the worst-case. Also, find a simple randomized algorithm which looks at $n^{0.893}$ leaves (in expectation). (This implies that the worst-case running time for any deterministic expected running time is $\Theta(n)$, while the randomized algorithm has expected running time $\Theta(n^{0.893})$ for any distribution.)

4. **Card Guessing:** Suppose a friend has a deck of n distinct cards (placed face-down in a pile), and you try to guess the card at the top of the pile. After your guess, the top card is turned over, so you can see whether you are correct. You then try to guess the next card, and so on until you go through the entire pile. Since you lack psychic powers, a reasonable strategy might be to guess cards at random.

   (a) If you have a terrible memory (so you cannot remember the cards you have already seen), each time you guess one of the n cards, chosen uniformly from all the possibilities. How many times do you expect to guess correctly?

   (b) If you have a *perfect* memory, so each time you only guess uniformly at random from one of the cards still in the pile, how many times do you expect to guess correctly?

5. **Max-Graph-3-Coloring:** In a graph coloring, an edge is *satisfied* if its two endpoints receive different colors. For any graph, let c^* be the maximum number of edges that can be satisfied by a 3-coloring. Give a simple randomized algorithm that computes a $3/2$ approximation to c^*.